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Self-similar flow behind a spherical shock wave in a non-ideal dusty gas under a gravitational field: Isothermal flow

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Abstract

Similarity solutions are obtained for one-dimensional unsteady isothermal flow of a dusty gas behind a spherical shock wave with time dependent energy input. The dusty gas is assumed to be a mixture of non-ideal (or perfect) gas and small solid particles, in which solid particles are continuously distributed. It is assumed that the equilibrium flow-conditions are maintained, and the viscous stress and heat conduction of the mixture are negligible. The medium is taken to be under the influence of the gravitational field due to a heavy nucleus at the origin (Roche model). The total energy of the flow-field behind the shock is increasing. The effects of an increase in the mass concentration of solid particles, the ratio of the density of the solid particles to the initial density of the gas, the gravitational parameter (or shock Mach number), and the parameter of non-idealness of the gas in the mixture, are investigated. It is shown that due to presence of gravitational field the isothermal compressibility of the medium and the flow-variables increases and the shock strength decreases. A comparison has also been made between the medium with and without gravitational field. The shock waves in dusty medium can be important for description of star formation, shocks in supernova explosions, etc.

Keywords: Shock wave; Self-similar solution; Dusty gas; Gravitational field; Roche model

1. Introduction

The study of shock waves in a mixture of a gas and small solid particles is of great importance due to its applications to nozzle flow, lunar ash flow, bomb blast, coal-mine blast, under-ground, volcanic and cosmic explosions, metalized propellant rocket, supersonic flight in polluted air, collision of coma with a planet, description of star formation, particle acceleration in shocks, formation of dusty crystals and many other engineering problems (see Pai et al., 1980; Higashino and Suzuki, 1980; Miura and Glass, 1983; Gretler and Reginfelder, 2002; Gretler and Regenfelder, 2005; Popel and Gisko, 2006; Vishwakarma and Nath, 2006, 2009; Sommerfeld, 1985, Elperin et al., 1987; Nath, 2010, 2012). Shock waves often arise in nature because of a balance between wave breaking non-linear and wave damping dissipative forces (Zel'dovich and Raizer, 1967). Collisional and collisionless shock waves can appear because of friction between the particles and wave–particle interaction respectively (Sagdeev, 1966). Miura and Glass (1985) obtained an analytical solution of a planar dusty gas flow with constant velocities of the shock and the piston moving behind it. As they neglected the volume occupied by the solid particles mixed into the perfect gas, the dust virtually had a mass fraction but no volume fraction. Their results reflect the influence of the additional inertia of the dust upon the shock propagation.

In order to get some essential features of the shock propagation, small solid particles are considered as a pseudo-fluid, and the mixture at velocity and temperature equilibrium with a constant ratio of specific heats (Pai, 1977). For this gas particle mixture to be treated as a

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so-called idealized equilibrium gas (Geng and Groenig, 1980), it is necessary to consider the particle diameter much smaller than a characteristic length of the flow-field and their number density is small in relation to that of the gas particles. The Brownian motion of the solid particles is negligibly small. No deformations and no phase changes of the solid particles occur. Gas and solid particles are chemically inert. In this case, we may assume that the viscous stress and heat conduction of the medium are negligible (Pai et al., (1980); Higashino and Suzuki, 1980).

At extreme conditions that prevail in most of the problems associated with shock waves, the assumption that the gas is ideal is no longer valid. In recent years, several studied have been performed concerning the problem of shock waves in non-ideal gases, in particular, by Anisimov and Spiner (1972), Ranga Rao and Purohit (1976), Wu and Roberts (1996), Madhumita and Sharma (2004), Vishwakarma and Nath (2006, 2009) and Nath (2007, 2012) among others.

By employing integral method of Laumbach and Probstin (1969) and Naidu et al. (1985) obtained an approximate analytic solution for a self-similar flow behind a strong shock wave propagating outward in particle-laden gas with variable total energy. Vishwakarma and Nath (2006, 2009), Gretler and Reginfelder (2002) and Gretler and Regenfelder (2005) obtained similarity solutions for shock waves in radiating and non-radiating dusty gas. Recently Vishwakarma and Nath (2010) and Nath (2010, 2012) obtained the similarity or non-similarity solutions for shock waves in a radiating and non-radiating rotating dusty gas. But these studies do not include the effect of gravitational force to the flow of dusty gas.

In all of the works mentioned above, the influence of gravitational field on the medium is not considered by any of the author in case of isothermal flow of a dusty gas behind a shock wave with time dependent energy input. The gravitational force has considerable effect on many astrophysical problems. Carrus et al. (1951) have studied the propagation of shock waves in a gas under the gravitational attraction of a central body of fixed mass (Roche model) and obtained the similarity solutions by numerical method. Rogers (1957) has discussed a method for obtaining analytical solution of the same problem. Ojha et al. (1998) have discussed the dynamical behavior of an unsteady magnetic star by employing the concepts of the Roche model in an electrically conducting atmosphere. Singh (1982) has studied the self-similar flow of a non-conducting perfect gas, moving under the gravitational attraction of a central body of fixed mass, behind a spherical shock wave assuming the total energy content between the inner expanding surface and the shock front to be increasing with time.

The purpose of this study is to obtain the similarity solutions for one-dimensional unsteady isothermal flow of a dusty gas behind a spherical shock wave with time dependent energy input in presence of a gravitational field. The medium is assumed to be under the gravitational field due to a heavy nucleus at the origin (Roche model). The unsteady model of Roche consists of a dusty gas distributed with spherical symmetry around the nucleus having large mass $\overline{m}$. The gravitational effect of the mixture itself is assumed to be negligible in comparison with the attraction of the heavy nucleus. In order to obtain the similarity solutions of the problem the density of the undisturbed medium is taken to be constant. The total energy of the flow-field behind the shock is increasing with time. This increase can be obtained by the pressure exerted on the fluid by an inner expanding surface or a piston (Rogers (1957)). The effects of a change in the value of the parameter of non-idealness of the gas in the mixture $\overline{b}$, the mass concentration of solid particles in the mixture K_p, the ratio of the density of solid particles to the initial density of the gas G_1 and the variation of the parameter of gravitation $\frac{1}{\alpha}$ (or shock Mach number M) are obtained. It is shown that due to presence of gravitational field the isothermal compressibility of the medium and the flow-variables increases and the shock strength decreases. A comparative study between the solutions of medium with or without gravitational effects is also made.

2. Basic equations and boundary conditions

The equations of motion for one-dimensional, unsteady isothermal and spherically symmetric flow of the mixture of non-ideal gas and small solid particles, in presence of a gravitational field are (c.f. Rogers, 1957; Zhuravskaya and Levin, 1996; Korobeinikov, 1976; Laumbach and Probstein, 1970; Nath, 2011)

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{2u\rho}{r} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\overline{G}\overline{m}}{r^2} = 0, \tag{2}$$

$$\frac{\partial T}{\partial r} = 0, \tag{3}$$

where ρ, u, p and T are the density, the flow velocity, the pressure and the temperature of the mixture respectively. $\overline{m}$ is the mass of the heavy nucleus at the centre, $\overline{G}$ is the gravitational constant, r is the radial distance and t is the time.

We assume that a spherical shock wave is propagating outward from the centre of symmetry in the undisturbed medium (mixture of a non-ideal (or perfect) gas and small solid particles) with constant density, under the gravitational force. The flow variables immediately ahead of the shock front are

$$u = 0, \tag{4}$$

$$\rho = \rho_1 = \text{constant}, \tag{5}$$

$$p = p_1 = \frac{\rho_1 \overline{m}\,\overline{G}}{R}, \tag{6}$$

where R is the shock radius.