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Analytic and numerical treatment of motion of dust grain particle around triangular equilibrium points with post-AGB binary star and disc

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Abstract

This paper investigates the motion around the triangular equilibrium points, of a passively gravitating dust particle in the gravitational field of a low-mass post-AGB binary system, surrounded by circumbinary disc. The two bodies of the binary are modeled as a triaxial star and a radiating-oblate star. Due to small deviation of disc stars on circular orbits, we have assumed that the Coriolis and centrifugal forces of the stars are slightly perturbed. The triangular equilibrium points of the particle are found. These points are defined by, triaxiality of the primary star, oblateness and radiation of the secondary one and the gravitational potential from the disc mass. Further, when the disc mass increases, the particle moves nearer to the stars and farther away from the disc. In general, these equilibrium points are linearly stable when $\mu < \mu_C$; where μ is the mass ratio and μ_C is the critical mass function, defined by the parameters of the system. The effects of each of these parameters on the size of the stability region are stated, and the periodic motion around the stable points is examined. It is seen that the orbits are ellipses, and the orientation, eccentricities, lengths of the semi-major and semi-minor axes are influenced by the parameters of the problem. In particular, for our numerical linear stability analysis, we have taken an extremely depleted pulsating star, IRAS 11472-0800 as the post-AGB triaxial star, with a weakly-radiating young white dwarf star; G29-38 as the secondary. For this system, the stability result of the triangular points comes out different. Here, $\mu_C < \mu$ throughout the entire range of the mass ratio and the critical mass function. Hence, the triangular equilibrium points are unstable. The stability of the orbits is tested using the Poincaré surfaces of section (Pss). The region of stability is controlled by the introduced parameters and the Jacobi constant.

Keywords: R3BP; Triaxial; Post-AGB stars; Circumbinary disc

1. Introduction

The restricted three-body problem (R3BP) describes the motion of an infinitesimal mass moving under the gravitational effects of two finite masses, called primaries, which move in circular orbits around their center of mass on account of their mutual attraction and the infinitesimal mass not influencing the motion of the primaries. The study of the R3BP is of great theoretical, practical, historical and educational relevance, and in its many variant, has had important implications in several scientific fields, including among others, celestial mechanics, galactic dynamics, chaos theory and molecular physics.

The investigations of problems involving oblateness and triaxiality, of one or both primaries have been extensively studied in past years. We start with the study conducted by Khanna and Bhatnagar (1999), in which they assumed the first primary to be an oblate spheroid and the second

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one a triaxial rigid body. Later, Sambhus and Sridhar (2000) investigated stellar orbits in triaxial clusters around black holes in galactic nuclei while Sharma et al. (2001) took both primaries as triaxial rigid bodies, under the frame of the R3BP. AbdulRaheem and Singh (2006) studied the case when both primaries are oblate spheroid and are radiating as well under effect of small perturbations in the Coriolis and centrifugal forces. Singh and Begha (2011) considered the bigger primary as a triaxial body and the smaller one as an oblate spheroid, when the Coriolis and centrifugal forces are perturbed. Singh and Mohammed (2012) examined the Robe's circular R3BP, when the first primary is an oblate spheroid and the second one, is a triaxial rigid body. Rambaux (2013) studied the rotational motion of the second largest body "Vesta" of the main asteroid belt, based on its orbital perturbations and its large triaxial shape.

Some studies of our planetary systems have revealed some disc of dust particles, which are regarded as young analogues of the Kuiper Belt in our Solar System. In stellar systems, this phenomenon is also valid. Out of an observed 69 A3-F8 main sequence binary star systems, nearly 60% showed dust discs surrounding binary stars. Greaves et al. (1998) found a dust ring around a nearby star, *e Eridani*.

Stars of low to intermediate main-sequence mass, up to about $6-8\ M_{\odot}$ will eventually evolve up the asymptotic giant branch (AGB, for short) stars (Iben and Renzini, 1983). These stars are surrounded by dust-rich shells of matter caused by their own stellar wind, which absorb the radiation coming from the central object and re-emit it in the far infrared. AGB stars are one of the largest distributors of dust into the interstellar medium due to their high mass-loss rates in combination with an effective dust condensation. During the latest phases of AGB star evolution, the mass loss often makes the star becomes invisible in the optical and in the part near infrared. On departure from the AGB, a fundamental transition in the mass loss process is taking place, changing from spherically symmetric outflow on the AGB, to axisymmetric or point symmetry geometries. Olivier et al. (2001) studied dust-enshrouded AGB stars in the solar neighborhood. Feast et al. (2003) investigated the case for an asymmetric dust around a C-rich AGB star while Piovan et al. (2003) investigated Shells of dust around AGB stars. Oxygen-rich AGB stars with optically thin dust envelopes was studied by Heras and Hony (2005) while dust cloud formation in stellar environments with emphasis on the two-dimensional models for structure formation around AGB stars was investigated by Woitke and Niccolini (2005). Zhao-Geisler et al. (2012) worked on dust and molecular shells in AGB stars. The dust plays a crucial role for the formation and acceleration of the dense wind. By providing the seed particles for interstellar grains, AGB stars contribute to the chemical evolution of the interstellar medium (ISM) and facilitate further star and planet formation.

Post-AGB stars are evolved stars with low to intermediate mass ($0.8-8.0\ M_{\odot}$) progenitors, that have left the AGB but not yet become the central stars of planetary nebulae (Van Winckel, 2003). The heavy mass-loss that occurred on the AGB has ended and the dust envelope that surrounds the star, as a result of this mass-loss, expands away from the star, decreasing in temperature while the temperature of the central star increases. During the subsequent phases of the evolution, the events ultimately lead to the formation of a planetary nebula, and subsequently evolve into a white dwarf. Vassiliadis and Wood (1994) studied the post-asymptotic giant branch evolution of, low-to intermediate-mass stars. In their studies, they presented results for post-AGB phases of stellar evolutionary sequences, complete from main-sequence phase, through the AGB phase, and on into, the planetary nebula and white dwarf regimes.

The young white dwarf Giclas 29-38 (WD 2326+049; McCook and Sion, 1987; G29-38 hereafter) having an assumed mass of $0.69\ M_{\odot}$ has garnered intense interest since Zuckerman and Becklin (1987) discovered infrared emission from this object far in excess of the photosphere and proposed that the excess could be due to a brown dwarf companion. This suggestion inspired discussion of brown dwarfs as white dwarf companions (Stringfellow et al., 1990), oscillating brown dwarfs (Marley et al., 1990), and other possible cool companions that could explain the excess (Greenstein, 1988). Kuchner et al. (1998) observed that the infrared excess of G29-38 is not due to a single orbiting companion and this supports the hypothesis that source of the near-infrared excess is not a cool companion but a dust cloud. The presence of a dust cloud around G29-38 is likely related to the anomalous presence of metals in its photosphere. Reach et al. (2005) investigated the dust cloud around G29-38 using powerful instruments of Spitzer Space Telescope. These observations support the idea that a relatively recent disruption of a comet or asteroid created the dust cloud around the star G29-38 and suggests that the material around G29-38 is not accreted ISM but rather is indigenous to the star.

Circumbinary disc are commonly observed around post-AGB systems and are known to play an important role in their evolution. Trilling et al. (2007) detected debris discs in many main-sequence stellar binary systems using the *Spitzer Space Telescope*. Van Winckel et al. (2009) observed that circumbinary discs around a post-AGB star, which should have been formed during a strong binary interaction, have major impact on the evolution of binary system, as their effects could shorten the AGB life time, judging from the measured orbits and mass function. IRAS 114712-0800, is an example of a highly evolved post-AGB star of spectral type F. It is a strongly depleted binary pulsating star surrounded by a stable circumbinary disc (Van Winckel et al., 2012). Hence, the study of the R3BP when the finite masses are surrounded by disc or belt is of great importance.