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Advances in Space Research 50 (2012) 371–384

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ADVANCES IN SPACE RESEARCH
(a COSPAR publication)

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An alternative computation of a gravity field model from GOCE

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Received 17 January 2012; received in revised form 11 April 2012; accepted 15 April 2012
Available online 23 April 2012

Abstract

GOCE is the first satellite with a gravitational gradiometer (SGG). This allows to determine a gravity field model with high spatial resolution and high accuracy. Four of the six independent components of the gravitational gradient tensors (GGT) are measured with high accuracy in the so-called measurement band (MB) from 5 to 100 mHz by the GOCE gradiometer. Based on more than 1 year of GOCE measurements, two gravity field models have been derived. Here, we introduce a strategy for spherical harmonic analysis (SHA) from GOCE measurements, with a bandpass filter applied to the SGG data, combined with orbit analysis based on the integral equation approach, and additional constraints (or stabilization) in the polar areas where no observation is available due to the orbit geometry. In addition, we combined the GOCE SGG part with a set of GRACE normal equations. This improves the accuracy of the gravity field in the long-wavelength parts, due to the complementarity of GOCE and GRACE. Comparison with other models and with external data shows that our results are rather close to the GPS-levelling data in well-selected test regions, with an uncertainty of 4–7 cm, for truncation at degree 200.

Keywords: Satellite gradiometry; Gravity gradiometry; Satellite-to-satellite tracking; GOCE; Gravity field; Variance component estimation

1. Introduction

GOCE (Gravity field and steady-state Ocean Circulation Explorer) launched in March 2009, has shown its high performance, (Pail et al., 2011; Rummel et al., 2011). It is the first mission of the European Space Agency's "Living Planet Programme". Its objectives are the determination of the static Earth's gravity field with an accuracy of one part per million of "*g*" (1 mGal) and of the global geoid surface with an accuracy of 1–2 cm. Both objectives shall be achieved with a spatial resolution of about 100 km half-wavelength, corresponding to a spherical harmonic expansion truncated at degree and order (d/o) 200 (European Space Agency, 1999; Rummel et al., 2002; Johannessen et al., 2003).

The core instrument of GOCE is an electrostatic gravity gradiometer (EGG). It consists six accelerometers mounted pairwise on three orthonormal axes. The accelerations at the location of each accelerometer are observed. GOCE is the first satellite capable of measuring the second derivatives of the gravitational potential, i.e., of the elements of the second-order gravitational gradient tensor V_{ij}, with $V_{ij} = \frac{\partial^2 V}{\partial x_i \partial x_j}$, where x_i, x_j define the axes of the gradiometer. The measuring principle is differential accelerometry (Stummer et al., 2011). After elimination of the rotational effects from the differential mode accelerations, the tensor components V_{xx}, V_{yy}, V_{zz} and V_{xz} can be deduced with high precision and the components V_{xy} and V_{yz} with much lower precision. All components are expressed in the satellite-fixed gradiometer reference frame (GRF) (see Gruber et al., 2010). For a detailed description of the derivation of the gravitational gradients from the differential mode accelerations, the so-called Level-1 data processing, the reader is referred to Frommknecht et al. (2011), Stummer et al. (2011). The gradiometer principle was chosen for this mission in order to counteract, to some extent, the attenuation of the gravitational field at satellite altitude by

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http://dx.doi.org/10.1016/j.asr.2012.04.018

observing differentiated quantities. An orbit altitude of only 265 km is maintained by continuous drag compensation in flight direction by ion thrusters (Floberghagen et al., 2011).

For the determination of a gravity field model based on GOCE measurements, the long-wavelength features are derived mainly from the orbits, i.e., from satellite-to-satellite tracking (SST) of GOCE to the GPS satellites. The so-called kinematic orbits are derived geometrically, i.e. without use of orbital mechanics. They are reconstructed with cm-precision (Bock et al., 2011; Visser et al., 2010). They are the basis for gravity field recovery from the SST part. The gradiometer measurements are sensitive to the short spatial scales of the gravity field, approximately at scales between 200 and 100 km or expressed in a spherical harmonic series, between d/o 100 and 200. By combining the SST and SGG parts, a gravity field model can be recovered with good accuracy in both the long- and short-wavelengths. The gradiometer measurements can also be combined with GRACE (Tapley et al., 2004), for a combined satellite-only solution. For the very low harmonics, e.g. below d/o 5, where both GRACE and GOCE are less sensitive, satellite laser range to satellites such as LAGEOS-1 and 2 (Lerch et al., 1982) should be considered for the combination.

Due to the GOCE orbit inclination of about 96.5°, there are polar gaps at the north and south pole, where no measurement is available. In the case of a spherical harmonic analysis, some real or pseudo information is necessary in order to eliminate this polar gap deficiency (Metzler and Pail, 2005). Variance components of the various observation types are needed as weights in a combination. They play a very important role for the combined solution, see Brockmann and Schuh (2010). The two GOCE gravity models discussed here are solutions of a combination of gradiometer measurements and GOCE kinematic orbits for a GOCE solution, and of also GRACE measurements for a GOCE–GRACE combined solution, in addition to some constraint applied to the polar areas.

There are three types of GOCE gravity field model available, see Pail et al. (2011). They are derived for the European Space Agency (ESA) by the GOCE High Level Processing Facility (HPF) with three different approaches, namely the time-wise approach (TIM), the direct approach (DIR) and the space-wise approach (SPW). For detailed description of the approaches the reader is referred to Pail et al. (2011), Metzler and Pail (2005) and Migliaccio et al. (2004). In addition to these methods for gradiometer data processing, Rummel (1986) suggested the so-called rotational invariant approach, which was modified and applied by Baur et al. (2007) and Yu et al. (2010). Another approach for GOCE data processing, applied for quick-look gravity field analysis is the so-called semi-analytical method. It is based on Hill's theory and was studied in Sneeuw (2000). It was implemented for quick-look analysis of real GOCE data by Mayrhofer et al. (2010). Recently two new GOCE models, i.e. the third generation gravity field models, became available. They are denoted as TIM3 and DIR3 in this paper. The observation data period of TIM3 is from November 1st, 2009 to April 17th, 2011, and that of DIR3 is from November 1st, 2009 to April 19th, 2011. The DIR3 model was computed not only with GOCE measurements, but also used GRACE observations for the period 24 February 2003 through 30 June 2009.

In this paper, we introduce a somewhat modified method for spherical harmonic analysis from GOCE measurements. Our strategy differs from the other GOCE-based solution in the following aspects: first, as shown e.g. in Rummel et al. (2011), it is known that the SGG observations are very precise in the MB. Therefore, a bandpass filter concentrated at the MB of $[5,100]$ mHz is applied to all four sensitive components in our solutions. A bandpass filter with a passband of $[10,125]$ mHz is applied to the three diagonal components of the DIR3 solution, and cascaded digital filters are applied to the four sensitive components for TIM3 solution. Second, in our solution the SST part is estimated up to d/o 150, using the integral equation approach applied to the GOCE kinematic orbits, while DIR3 uses GRACE and LAGEOS observations for the long- and medium-wavelength part employing the classical orbit perturbation approach with a solution up to d/o 160, and TIM3 uses GOCE kinematic orbits set up to d/o 120 by employing the energy balance approach. Third, the constraints on the polar areas are applied in different ways in TIM3, DIR3 and our solutions. TIM3 and DIR3 used the spherical cap stabilization (Metzler and Pail, 2005), whereas we use discrete values from an a priori gravity field model. Unlike Kaula regularization as applied in GOCO02S (Goiginger et al., 2011), TIM3 and DIR3, we compute our solutions without Kaula regularization but using a "soft" polar constraint in order to introduce some compensation in the polar areas.

This study presents our method of SHA with GOCE observation, and the combination of GOCE and GRACE. In Section 2, we present the methodology and theory of our GOCE data processing, including the observation model of SST and SGG, as well as the polar area constraint with pseudo-observations. The combination of different observation types is discussed in this section. In Section 3, a GOCE-based solution and a GOCE-GRACE combined solution are presented with a detailed descriptions. In Section 4, the models are analyzed by comparing them to other models and to independent data sets. We summarize this work and draw our conclusions in Section 5.

2. Observation model and combination

The Earth gravitational potential can be represented as the following spherical harmonic series (see Montenbruck and Gill, 2000, p. 57):

$$V = \frac{GM_\oplus}{R_\oplus}\sum_{n=0}^{N_{max}}\sum_{m=0}^{n}\left(\overline{C}_{nm}\overline{V}_{nm} + \overline{S}_{nm}\overline{W}_{nm}\right), \tag{1}$$