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GOCE orbit analysis: Long-wavelength gravity field determination using the acceleration approach

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Abstract

The restricted sensitivity of the Gravity field and steady-state Ocean Circulation Explorer (GOCE) gradiometer instrument requires satellite gravity gradiometry to be supplemented by orbit analysis in order to resolve long-wavelength features of the geopotential. For the hitherto published releases of the GOCE time-wise (TIM) and GOCE space-wise gravity field series—two of the official ESA products—the energy conservation method has been adopted to exploit GPS-based satellite-to-satellite tracking information. On the other hand, gravity field recovery from data collected by the CHAllenging Mini-satellite Payload (CHAMP) satellite showed the energy conservation principle to be a sub-optimal choice. For this reason, we propose to estimate the low-frequency part of the gravity field by the point-wise solution of Newton's equation of motion, also known as the acceleration approach. This approach balances the gravitational vector with satellite accelerations, and hence is characterized by (second-order) numerical differentiation of the kinematic orbit. In order to apply the method to GOCE, we present tailored processing strategies with regard to low-pass filtering, variance–covariance information handling, and robust parameter estimation. By comparison of our GIWF solutions (initials GI for "Geodätisches Institut" and IWF for "Institut für WeltraumForschung") and the GOCE-TIM estimates with a state-of-the-art gravity field solution derived from GRACE (Gravity Recovery And Climate Experiment), we conclude that the acceleration approach is better suited for GOCE-only gravity field determination as opposed to the energy conservation method.

Keywords: Kinematic positions; GOCE; GPS; Numerical differentiation; Empirical covariance function; Gravity field

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1. Introduction

Since autumn 2009, the Gravity field and steady-state Ocean Circulation Explorer (GOCE) satellite collects science data that are exploited to determine the static terrestrial gravity field. The spacecraft's core instrument is a three-dimensional gravity gradiometer. The gradiometer measurement bandwidth ranges from 5 mHz to 0.1 Hz (ESA, 1999), roughly corresponding to degrees and orders 30 to 250 in terms of the representation of the geopotential in spherical harmonics. Thus, for precise gravity field modeling gradiometry has to be complemented in the low-wavelength spectrum.

Due to the complementary sensor concepts, GOCE gradiometry may be combined with inter-satellite range and range-rate observations provided by the Gravity Recovery And Climate Experiment (GRACE) (Tapley et al., 2004). The models of the GOCO series (Pail et al., 2010), for instance, are a realization of this combination strategy. Alternatively, long-scale features can be extracted from GOCE orbit perturbations as sensed by the Global Positioning System (GPS), referred to as satellite-to-satellite tracking in the high-low mode (hl-SST). GOCE-only

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gravity field solutions are internally consistent (Baur et al., 2010) and constitute one of ESA's key objectives in the framework of GOCE data analysis (ESA, 1999).

GPS-based hl-SST has extensively been studied for gravity field recovery from the CHAllenging Mini-satellite Payload (CHAMP) mission (Reigber et al., 2002). In this context, several analysis approaches have been proposed. The energy balance method (e.g., Gerlach et al., 2003; Han et al., 2002; Weigelt, 2007) relates GPS-derived satellite positions and velocities to the gravitational potential. The Celestial Mechanics Approach (CMA) (Beutler et al., 2010a,b) exploits orbit perturbations via variational equations (e.g., Montenbruck and Gill, 2000). The short-arc method (Mayer-Gürr, 2006) is based on the formulation of Newton's equation of motion as a boundary value problem in combination with gravity field modeling over short periods. The acceleration approach balances the gravitational vector with satellite accelerations obtained from numerical differentiation of the kinematic orbit (Ditmar and van Eck van der Sluijs, 2004; Reubelt et al., 2003; Reubelt, 2009). Comparisons of these approaches revealed the energy balance method to be inferior to the other strategies (Ditmar and van Eck van der Sluijs, 2004; Löcher, 2010; Mayer-Gürr et al., 2005; Reubelt et al., 2012).

For some of their releases, the GOCE High-level Processing Facility adopted the energy balance principle to determine the long-wavelength gravitational spectrum by means of orbit analysis. In particular, to date the method is used for the GOCE-only time-wise (GOCE-TIM) gravity field series, one of the official ESA products (Pail et al., 2011). Analyzing kinematic GOCE satellite positions covering a period of two months (November and December 2009), Jäggi et al. (2011a) showed that the CMA outperforms results based on energy balance by a factor of $\sqrt{3}$.

Here we propose the acceleration approach to be used for the analysis of GOCE hl-SST measurements. Simulation experiments demonstrated that this method yields comparable improvement as obtained with the CMA (e.g., Ditmar and van Eck van der Sluijs, 2004; Reubelt, 2009). Our results on GOCE real data show that also the acceleration approach has the potential to improve the current GOCE-TIM models up to spherical harmonic degree (and order) $l \approx 25$.

2. Methods

2.1. Observation equation

The acceleration approach is based on Newton's equation of motion in the inertial space. Making use of the equivalence principle, the acceleration of a satellite in the terrestrial gravity field, $\mathbf{a}(t)$, is equal to the Earth's gravitational pull on the spacecraft, and hence the gradient of its scalar-valued counterpart, the geopotential $V(t)$. Furthermore, in inertial space the satellite acceleration can be represented as the second-order temporal derivative of the satellite position $\mathbf{x}(t)$. Consequently, the relation

$$\mathbf{a}(t) = \ddot{\mathbf{x}}(t) = \nabla V(t) \tag{1}$$

holds true.

In real-case environments, the acceleration of a satellite is not solely subject to the Earth's gravity field, but results from the superposition of various forces acting on the spacecraft. Gravitational disturbances include third-body accelerations $\mathbf{a}_\mathrm{b}(t)$, tidal accelerations $\mathbf{a}_\mathrm{t}(t)$, relativistic effects $\mathbf{a}_\mathrm{r}(t)$, and variations in gravitation caused by short-term fluctuations (atmosphere, oceans, hydrology, etc.) $\mathbf{a}_\mathrm{f}(t)$. We modeled these disturbances in terms of corrections to the total satellite acceleration; the models used for the computation of the corrections are listed in Table 1.

We would like to emphasize that the acceleration approach applied to Low-Earth Orbiters (LEOs) is largely insensitive to non-gravitational disturbances. Using a priori gravity field information for CHAMP data analysis, Ditmar et al. (2007) concluded that cleaning satellite accelerations from non-gravitational effects does not lead to superior spherical harmonic coefficient estimates; they attributed this behavior to different contamination of the observational components and to orbital "symmetry" (cf. Ditmar et al., 2007, Sect. 5). These findings were supported by Reubelt (2009) without the use of gravitational prior knowledge.

The low GOCE orbit implies that the satellite is more prone to atmospheric drag compared to the CHAMP spacecraft. This disturbing signal, however, is largely compensated by the drag-free control. As a consequence, the residual surface forces are small, and hence the conclusions derived from CHAMP also hold for GOCE. In order to prove this assumption numerically, we co-estimated non-gravitational disturbances in terms of empirical parameters (sub-daily biases). It turned out that this strategy led to no improvement for spherical harmonic coefficients recovery.

Parameterized in spherical harmonics, the geopotential $V(t)$ on the right hand side in Eq. (1) reads

$$V(\lambda,\varphi,r) = \frac{GM}{a}\sum_{l=0}^{L}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+1}\overline{P}_{lm}(\sin\varphi) \left[\bar{c}_{lm}\cos m\lambda + \bar{s}_{lm}\sin m\lambda\right] \tag{2}$$

(Heiskanen and Moritz, 1967). Therein (λ,φ,r) denote spherical polar coordinates with λ longitude, φ latitude and r distance from the geocenter. GM is the gravitational

Table 1
Background models used in order to account for gravitational disturbing effects.

	Model	Reference
Third-body accelerations	Moon and Sun	Petit and Luzum (2010)
Solid Earth tides	Yide-free	Petit and Luzum (2010)
Ocean tides	FES2004	Lyard et al. (2006)
Solid Earth pole tide	IERS	Petit and Luzum (2010)
Ocean pole tide	Desai	Desai (2002)
Atmospheric tides	N1	Biancale and Bode (2006)
Relativistic corrections	IERS	Petit and Luzum (2010)
Short-term variations	AOD1B	Flechtner (2008)