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Spectral line broadening by relativistic electrons in plasmas: Collision operator

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Abstract

In the present work we compute the real part of the impact collision operator for the electron broadening of ion lines in plasmas, taking into account relativistic effects in the dynamics of the perturbing electron. Specifically two relativistic effects are included: The modification of the trajectory due to non-Newtonian mechanics and the modification of the velocity distribution (Maxwell–Juttner). The results are compared to the non-relativistic case.

Keywords: Relativistic electrons; Relativistic trajectories; Collision operator; Real part of collision operator

1. Introduction

Line profiles and shifts are used to determine plasma parameters, especially in astrophysics where alternative methods (such as interferometry or Thomson scattering) are not possible. Doppler and pressure broadening are typically the two dominant mechanisms and we focus on the later. In a number of hot astrophysical plasmas, electrons may be energetic enough that their thermal energy K_BT can be comparable to the rest mass. For the extreme densities encountered in some astrophysical objects, pressure broadening could dominate; however for such objects the electrons may become relativistic due to the extreme temperatures and hence it makes sense to check the modifications to the pressure broadening by relativistic effects. Similarly in laser-produced plasmas very high densities may be achieved sometimes with rather modest temperatures, resulting in the dominance of Stark over Doppler broadening and sometimes with very high temperatures, resulting in relativistic electron velocities.

More specifically, plasma spectroscopy is used in a wide range of electron density from 10 particles per cm^3 (interstellar space) to 10^{25} particles per cm^3 (star interiors, inertial confinement fusion) and for temperatures between 10^7 K and 10^{10} K. In the present work we investigate the region corresponding to the particular conditions of plasma: high density and high temperature. Under these conditions, (electron–ion) collisions will be, throughout this work, assumed binary and the dynamics of the electrons will be treated relativistically. Furthermore, in this work we shall use the statistical classical mechanics (not the quantum statistical mechanics as in the Fermi–Dirac distribution) because the condition $\lambda_{th} = h/\sqrt{2\pi m_e K_B T} < N_e^{-1/3}$ (m_e is the electron mass, K_B is the Boltzmann constant, T is the temperature, λ_{th} is the De Broglie thermal length and N_e is the electrons density) is fulfilled in the stated density and temperatures ranges. For example, if $N_e = 10^{24}$ cm^{-3} and $T > 10^7$ K, it is easy to verify that this inequality holds. This condition means that the wave function extent (λ_{th}) associated with the electron is smaller than the mean distance ($\sim N_e^{-1/3}$) between two free electrons.

In the present work we focus on electron broadening in the impact approximation (Anderson, 1949; Griem et al., 1962). We thus revisit the standard semiclassical collision

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operator and take into account relativistic effects with respect to the trajectory. In many cases in line broadening, fast particles (typically electrons) are described by a collisional approach, while particles whose field variation on the inverse half width half maximum (HWHM) time scale are considered static and treated via a quasistatic microfield. For many applications, isolated lines are of great importance. Calculations of the broadening of such a line in a plasma are normally made using the impact approximation for electrons (Anderson, 1949) in the semi-classical version (Baranger, 1958a; Griem et al., 1962; Sahal-Bréchot, 1969b), as the ionic contribution is typically negligible. We only consider isolated lines, since within the impact approximation the collision operator for more complex cases is basically expressible in terms of this case.

This paper is organized in four sections. The second section derives the collision operator along the lines of the standard nonrelativistic framework, but with account for relativistic effects. The third section compares the relativistic and nonrelativistic operators and discusses the obtained results, while the last section summarizes the results.

2. Relativistic collision operator

As discussed above, electrons can be treated in the impact approximation. The validity of the impact approximations is that the collision delay τ, must be too small compared with the time interval with which we calculate the correlation function (Baranger, 1958a,b). τ can be written as:

$$\tau = \overline{\rho}/\overline{v} \tag{1}$$

where $\overline{\rho}$ is the typical impact parameter and $\overline{v}$ is the mean velocity of the collider:

$$\overline{v} = \left(\frac{2K_BT}{m_e}\right)^{1/2} \tag{2}$$

where m_e is the rest mass of the electron, and K_B is the Boltzmann constant. The validity of the impact theory is written as (Baranger, 1958a,b,c; Sahal-Bréchot, 1969a):

$$\omega\tau \ll 1 \tag{3}$$

An order of magnitude of $\overline{\rho}$ can be derived from the line width ω and is given by:

$$\overline{\rho}^2 = \frac{e^2\overline{r}}{\hbar\omega} \tag{4}$$

where $\overline{r}$ is the mean radius of the hydrogenoid atom related with the upper level principal quantum number n and the first Bohr orbit radius a_0 by:

$$\overline{r} = n^2a_0/Z \tag{5}$$

where Z is the spectroscopic charge number. By using the estimation of $\overline{\rho} = \left(\frac{3}{4\pi}\right)^{1/3}N_e^{-1/3}$ the validity condition of the impact approximation can be written as (Baranger, 1958b):

$$\frac{Z^3A}{n^6} \gg 1 \tag{6}$$

with:

$$A = \frac{2(2\pi m_eK_BT)^{3/2}}{N_eh^3} \tag{7}$$

where A is the number of the accessible quantum states for the electron. In the range of densities and temperatures considered in our study, this condition is well satisfied. Alternatively, one could extrapolate from well-known benchmark calculations (Hegerfeld and Kesting, 1988) to arrive at the same conclusion.[1]

Using the polar coordinates (r,α), with r the radial distance and α the polar angle, the hyperbolic trajectory of the perturber is given by:

$$\frac{\rho^2}{r\rho_0} = 1 + \varepsilon\cos\alpha \tag{8}$$

where ρ is the impact parameter, and ε is the eccentricity given by:

$$\varepsilon = \sqrt{1 + \left(\frac{\rho}{\rho_0}\right)^2} \tag{9}$$

The half-major axis ρ_0 of the hyperbola is related to the velocity v, and the spectroscopic charge number Z by:

$$\rho_0 = \frac{1}{4\pi\varepsilon_0}\frac{(Z-1)e^2}{m_ev^2} \tag{10}$$

where e is the electron charge, and ε_0 is the permittivity of free space.

Within the impact approximation the following result has been obtained (Alexiou, 1994):

$$\begin{aligned}\varphi_{int}(\zeta,-\zeta) = &-\frac{2\pi N_ee^4}{3\hbar^2}\frac{1}{(4\pi\epsilon_0)^2}\sqrt{\frac{2}{\pi}}\left(\frac{m_e}{K_BT}\right)^{3/2}\\ &\int_0^\infty v\exp\left(-\frac{m_ev^2}{2K_BT}\right)dv\int_{\varepsilon_{min}}^{\varepsilon_{max}}\Big[G_1(\zeta,\varepsilon)G_1(-\zeta,\varepsilon)\\ &+\frac{\varepsilon^2-1}{\varepsilon^2}G_2(\zeta,\varepsilon)G_2(-\zeta,\varepsilon)\Big]\frac{d\varepsilon}{\varepsilon}\end{aligned} \tag{11}$$

where ζ is the inelasticity parameter $\zeta = \omega_1\rho_0/v$ depending on the velocity v, the impact parameter ρ_0 and the Bohr frequency between the states connected by the collision ω_1. $\varepsilon_{\min}$ and $\varepsilon_{\max}$ are given, using (9), by:

[1] In the reference given, the impact approximation is shown to be excellent (better than 1.7% difference from exact calculations) for L_α in hydrogen for $T = 1$ eV and $N_e = 4\times10^{17}$ cm^{-3}. Hence using the $1/Z$ scaling for the matrix elements, linear density scaling and inverse square root scaling for T, we can extrapolate to $N_e = 10^{23}$ cm^{-3} and $T = 4\times10^9$ K and H-like Fe, so that the impact width would increase by a factor of 4, hence the inverse HWHM would decrease by that factor. However, due to the huge temperature, the collision duration would be about 600 times smaller and hence if electron collisions were impact at $T = 1$ eV and $N_e = 4\times10^{17}$ cm^{-3}, they will be even more so at $N_e = 10^{23}$ cm^{-3} and $T = 4\times10^9$ K.