Available online at www.sciencedirect.com

ScienceDirect

Advances in Space Research 54 (2014) 1326–1330

ADVANCES IN SPACE RESEARCH (a COSPAR publication)

www.elsevier.com/locate/asr

Shape modeling with family of Pearson distributions: Langmuir waves

Sonja Vidojevic *

IHIS—techno experts d.o.o., Batajnicki drum 23, 11080 Belgrade, Serbia

Available online 17 January 2014

Abstract

Two major effects of Langmuir wave electric field influence on spectral line shapes are appearance of depressions shifted from unperturbed line and an additional dynamical line broadening. More realistic and accurate models of Langmuir waves are needed to study these effects with more confidence. In this article we present distribution shapes of a high-quality data set of Langmuir waves electric field observed by the WIND satellite. Using well developed numerical techniques, the distributions of the empirical measurements are modeled by family of Pearson distributions. The results suggest that the existing theoretical models of energy conversion between an electron beam and surrounding plasma is more complex. If the processes of the Langmuir wave generation are better understood, the influence of Langmuir waves on spectral line shapes could be modeled better.

Keywords: Pearson system of distributions; In situ observations; Langmuir waves

1. Introduction

An effect of stochastically oscillating electric field of Langmuir waves on spectral line shapes and intensities of a radiator (e.g. hydrogen or deuterium) emerged in a quasi-static electric field is discovered experimentally and explained theoretically (for more details see Stamm et al. (2014), and references therein). Thus, the more realistic models of Langmuir waves are required for the research continuation in this field. One way to obtain them is to analyze actual measurements of Langmuir waves electric field in space plasmas. In this article we present distribution shapes of Langmuir waves electric field observed by the WIND satellite modeled by Pearson system of distributions.

The choice of the best-suited statistical distribution for empirical data modeling is not a trivial issue. Unless a sound theoretical background exists for selecting a particular distribution, one will usually resort to testing various candidates and select a distribution based on its fit to the observed data. While this is a legitimate strategy, it is more objective and efficient to define a sufficiently general family that can be used for this purpose. This approach has a long tradition in statistics, and resulted in various families of distributions, most notably Pearson's (Pearson, 1895).

A practical implementation of the distribution family and methods is applied to data sets of electric field of Langmuir waves measured by WIND satellite (Bougeret et al., 1995). The obtained distributions could yield to an improvement of existing theories and progress in a long-lasting problem in beam-plasma interaction mechanisms. Langmuir waves are one of the most typical and significant plasma waves observed in solar and space plasma. The interaction between the solar wind and fast energetic electron beams ($v \approx 0.03$ to $0.3\,c$) produces electrostatic Langmuir waves in each point of the electron beams trajectory. Generated Langmuir waves have a frequency of the local plasma frequency approximately (at 1 AU Langmuir waves have frequencies between 10 and 50 kHz). Some amount of energy of the Langmuir waves is converted into electromagnetic radiation – type III solar radio bursts. Although the electron beams, Langmuir waves and type III solar radio bursts have been observed and studied intensively by numerous authors (e.g. Gurnett and Anderson, 1976, 1981, 1998, etc.), the energy conversion mechanisms between them are still far from being well

* Tel.: +381 64 1730202.
E-mail address: sonja@matf.bg.ac.rs

0273-1177/$36.00 © 2014 COSPAR. Published by Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.asr.2013.12.027

understood. The most promising theories start with basic ideas of Ginzburg and Zhelezniakov (1958) introducing a nonlinear wave–wave interaction of Langmuir, ion-acoustic and electromagnetic waves. A second mechanism is the linear mode conversion in which a nearly monochromatic Langmuir wave incident on a density gradient partially reflects and partially converts into electromagnetic radiation near the plasma frequency (Yin et al., 1998). A third mechanism is quasi-mode method given by Yoon et al. (1994) where an induced scattering of forward propagating Langmuir waves of thermal ions induced backward propagating Langmuir waves. Neither of these theories can fully describe the observed phenomena. Therefore, it is of great importance to determine exact energy distribution of Langmuir waves, so the underlying physical processes might be recognized and incorporated in a theory.

To explain the observed Langmuir waves electric field distribution Robinson (Robinson, 1993) developed a theory, named stochastic growth theory (SGT), on the assumption that the Langmuir waves growth rate is randomly fluctuating. This hypothesis is based on the fact that waves and electrons are interacting in an inhomogeneous plasma environment. Assuming that the effective number of growth rate fluctuations is large enough, the central limit theorem of statistics can be applied. A consequence of the SGT is that the probability distribution of the logarithm of wave energy density should be Gaussian. Langmuir waves in the Earth's electron foreshock observed by the CLUSTER spacecraft have been statistically studied by Musatenko et al. (2007). They showed that the observed distributions for the logarithm of the wave intensities belong to Pearson system of distributions rather than being normal. This disagreement with the SGT prediction could be a result of an insufficient number of growth rate fluctuations in the typical Earth's electron foreshock conditions, so that the central limit theorem can not be applied. To see if the distribution for type IIIs is similar to the one in the Earth's bow shock, a set of 36 events encompassing 16 years of in situ WIND spacecraft observations is selected. For all 36 events electron beam, Langmuir waves and type III bursts are simultaneously detected. The preliminary data set analysis is presented in Vidojevic et al. (2010, 2012). In the present article the more accurate numerical technics are used to calculate distribution parameters of Langmuir waves electric field and the detailed practical implementation of Pearson system of distributions and numerical technics is given.

The aim of this study is to obtain an accurate empirical model of Langmuir waves electric field distribution from the observations using numerical methods. For this purpose, a high-quality data base of 36 events from the satellite WIND observations is selected. The capability and power of Pearson system in statistical examination of empirical data is demonstrated in general and an accurate model of electric filed distribution of Langmuir waves associated with type III solar bursts is obtained.

2. Pearson's system of distributions

The Pearson distributions are a widely used family of distributions to approximate empirical data, with a wide diversity of distribution shapes. The variety of shapes offered by this family includes unimodal, bimodal, U-shaped, J-shaped and monotone probability distribution functions, which may be symmetric and asymmetric, concave and convex, with smooth, abrupt, truncated, long, medium or short tails. On one hand, the ability of Pearson distributions to take this great diversity of shapes is responsible for wide application in actual modeling of measurements, and, on the other hand, the estimation of the distribution parameters requires only first four central moments calculated from the measurements minimizing the error propagation. The basic properties of the family members are discussed and numerical procedures for determining appropriate parameters using maximum likelihood estimation and method of moments are introduced.

In the late 19th century and early 20th, in a series of articles Karl Pearson introduced his system of probability distributions (Pearson, 1895, 1901, 1916). He defined this distribution system by the following first order ordinary differential equation for the probability density function $f(x)$:

$$-\frac{f'(x)}{f(x)} = \frac{a+x}{c_0 + c_1 x + c_2 x^2} \quad (1)$$

where a, c_0, c_1 and c_2 are four real independent parameters. The form of the solution of this differential equation depends on the value of these parameters, resulting in several distribution types.

The classification of distributions in the Pearson system is entirely determined by the first moment (mean–μ_1) and the next three central moments (variance–μ_2, skewness–μ_3 and kurtosis–μ_4). Pearson proposed two dimensionless parameters, i.e. the two moment ratios associated with the square of the skewness (β_1) and kurtosis (β_2):

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}, \quad \beta_2 = \frac{\mu_4}{\mu_2^2}. \quad (2)$$

These two parameters characterize the asymmetry and the peakedness of the distribution, respectively, and entirely determine the type of the Pearson distribution system through one parameter, κ, defined as:

$$\kappa = \frac{\beta_1(\beta_2+3)^2}{4(2\beta_1 - 3\beta_1 - 6)(4\beta_2 - 3\beta_1)}. \quad (3)$$

For $\kappa < 0, 0 < \kappa < 1$ and $\kappa > 1$, the distributions are called type I, type IV and type VI, respectively. These three cases are known as "the main types" because they occupy areas in the (β_1, β_2) space, contrary to the other types which are represented by lines or points. Type III ($\kappa = \pm\infty$) lies on the boundary between type I and type VI. Type V ($\kappa = 1$) lies on the boundary between type IV and type VI. If $\kappa = 1$, an additional condition is needed for the classification. The distribution is classified as type II if $\beta_1 = 0$