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# Optimal $H_\infty$ robust output feedback control for satellite formation in arbitrary elliptical reference orbits

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**Abstract**

A two degree-of-freedom signal-based optimal $H_\infty$ robust output feedback controller is designed for satellite formation in an arbitrary elliptical reference orbit. Based on high-fidelity linearized dynamics of relative motion, uncertainties introduced by non-zero eccentricity and gravitational J2 perturbation are separated to construct a robust control model. Furthermore, a distributed robust control model is derived by modifying the perturbed robust control model of each satellite with the eigenvalues of the Laplacian matrix of the communication graph, which represent uncertainty in the communication topology. A signal-based optimal $H_\infty$ robust controller is then designed primarily. Considering that the uncertainties involved in the distributed robust control model have a completely diagonal structure, the corresponding analyses are made through structured singular value theory to reduce the conservativeness. Based on simulation results, further designs including increasing the degrees of freedom of the controller, modifying the performance and control weighted functions, adding a post high-pass filter according to the dynamic characteristics, and reducing the control model are made to improve the control performance. Nonlinear simulations demonstrate that the resultant optimal $H_\infty$ robust output feedback controller satisfies the robust performance requirements under uncertainties caused by non-zero eccentricity, J2 perturbation, and varying communication topology, and that 5 m accuracy in terms of stable desired formation configuration can be achieved by the presented optimal $H_\infty$ robust controller. In addition to considering the widely discussed uncertainties caused by the orbit of each satellite in a formation, the optimal $H_\infty$ robust output feedback control model presented in the current work considers the uncertainties caused by varying communication topology in the satellite formation that works in a cooperative way. Other new improvements include adopting a new method to more accurately describe and analyze the effects of the higher-order J2 perturbation, combining all the uncertainties into a diagonal structure, and utilizing a structured singular value to synthesize and analyze the controller.


*Keywords:* Satellite formation flying; Optimal $H_\infty$ robust control; Output feedback control; Graph theory

## 1. Introduction

Satellite formation flying considers a group of satellites flying relatively close to one another to share one common payload and accomplish missions cooperatively. It has many potential advantages over conventional monolithic satellites, such as lower launch cost, higher launch flexibility, improved observation efficiency, greater system reliability, easier expandability, and enhanced survivability. This attracts many missions to consider a formation flying system with specified configurations instead of a large single satellite (Xu et al., 2007).

As one of the most crucial problems in determining the success of satellite formation missions, formation control has drawn a considerable amount of research

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effort. The Clohessy–Wiltshire (C–W) equation is most widely used in satellite formation control problems, where an unperturbed circular reference orbit is considered; that is, nonlinearities and external disturbances (such as the gravitational J2 perturbation) are not taken into account (Clohessy and Wiltshire, 1960). Thus, the resulting reference trajectory is not an invariant manifold for the satellite dynamics. As a result, a control system based on the unperturbed dynamics must compensate for the deviation of the satellite motion from the predicted trajectory, which may result in unacceptable fuel consumption and poor performance during formation reconfiguration (Serrani, 2003). For problems dealing with maintaining satellite formation, the control efforts required to overcome the J2 perturbation are proportional to satellite inclination (Xu et al., 2007). Thus, uncertainty rejection techniques (such as the Lyapunov method, sliding model control, adaptive control, robust method, etc.) have been applied in formation control design problems. Compared to other methods, the robust control method is less sensitive to noise and unmodeled dynamics and provides designers with a more systematic approach for analyzing and synthesizing control systems with both parametric and unmodeled uncertainties.

Serrani (2003) designed a robust coordinate decentralized controller for satellite formation with a leader–follower structure, where the desired trajectory of the leader was considered to be circular for simplicity. Serrani first employed an internal model of reference trajectories and gravitational disturbance to autonomously generate the control input required to offset the tracking error caused by a persistent disturbance. The generated feedforward control input rendered the desired formation pattern an invariant manifold for the formation dynamics. A robust error feedback controller was then designed, which stabilizes the resulting invariant manifold with the desired domain of attraction. Wu et al. (2006) considered the J2, J3, J4, and drag perturbations as bounded noises and established a linear perturbed relative motion model based on the linear C–W equation. They then designed a state feedback formation–maintaining controller that uses the $H_\infty$ norm of the transfer function from the disturbance to the regulated output as a performance index. Based on this previous research, Wu et al. (2006) further designed an $H_\infty/H_2$ mixed robust output feedback controller to restrain various perturbations for maintaining satellite formation. The designed controller with regional pole placement constraints considered both disturbance rejection aspects and LQG aspects. Dang and Zhang (2012) designed a satellite formation controller using a $\mu$-synthesis method for the Inner-Formation Gravity Measurement Satellite System (IFGMSS). The gravitational force of the inner satellite and varying mass of the outer satellite cause parametrical uncertainties, which render the C–W equation insufficient. Also, considering other uncertainties introduced by thrusters, they studied the robust control method based on the $\mu$-synthesis. Their $\mu$-synthesis robust controller was less conservative and improved the control efficiency compared with the conventional $H_\infty$ robust controller.

Research on satellite formation control is much more developed for circular reference orbits than for elliptical reference orbits. However, depending on the mission requirements, a satellite formation can sometimes implement more specific and effective missions flying on an elliptic reference orbit rather than on a circular orbit. Thus, along with the large amount of current literature on satellite formation control for circular reference orbits, some scholars have researched formation control for elliptical reference orbits.

Hu and Ng (2007) developed a robust control method for a two-spacecraft formation flying system. The state space form of the relative motion model was established by detaching the time-varying perturbation (the gravitational J2 disturbance force, etc.) from the original system. An additional robust control quantity was then designed to restrain the corresponding separated perturbations. The stability was finally verified through a Lyapunov analysis method. Lim et al. (2011) considered a relative position-control problem for a satellite formation system in an elliptical orbit, where the angular rate and acceleration were supposed to be bounded unknowns. Considering saturation of the control force, they designed a static state-feedback controller. In order to determine the parameters of the controller, they specified three invariant sets and a Lyapunov function and verified the stability conditions through an LMI approach. By collecting the non-zero eccentricity, varying semi-major axis and the J2 perturbation into parametric uncertainties, and collecting atmospheric drag and actuator/sensor noise to bounded functional uncertainties, Xu et al. (2007) first established the relative motion model for satellite formation in an arbitrary reference orbit, where the nonlinear terms are also considered. Thus, they designed a $\mu$-synthesis robust state feedback controller to restrain those uncertainties and achieved the required fuel consumption for formation maintenance by adding an additional weighted matrix in the control quantity channel. The proposed $\mu$-synthesis robust controller was compared with an LQR controller and a sliding mode controller for validation. Since Hu and Ng (2007) directly separated all the effects caused by the J2 perturbation to a multiplicative uncertainty; their robust control model is relatively conservative. In order to reduce the conservativeness, in the present work, we first separate the first-order J2 perturbation from high-fidelity linearized dynamics of relative motion and then utilize a relatively accurate multiplicative uncertainty whose magnitude is obtained from nonlinear simulations to describe the effect of the higher-order J2 perturbation.

A proper communication relationship between satellites is a critical factor that determines whether the formation can successfully conduct cooperative missions. In this paper, the communication relationship is the information exchanges between satellites in terms of navigation and controls. The graph theory, which is typically used to