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Long term evolution of Molniya orbit under the effect of Earth's non-spherical gravitational perturbation

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Abstract

A double resonance model is applied to study the long term evolution of a Molniya orbit, which is highly elliptical ($e \geqslant 0.7$), critically inclined ($i \approx 63.4^\circ$), and in the state of the 2:1 mean motion resonance with the Earth rotation. The dynamics of a Molniya orbit can be divided into three kinds: short (12 h), intermediate (several years) and long (several centuries) period motions, with the latter two studied in this paper. The J_2 and J_{l2} ($l = 2, 3, \ldots, 8$) harmonics are modelled, based on a careful selection. The analytic solution for the intermediate period motion is obtained, a first integral, $\bar{I}_3$, for the long period motion is derived analytically, and the phase structures are obtained by the level curves of $\bar{I}_3$. Three types of the phase structures, depending on the equilibria and stabilities, are observed when the Hamiltonian constant varies. Compared with the near circular 12-h satellite orbits and with the critically inclined orbits without mean motion resonance with the Earth rotation, the features of the Molniya orbits are discussed in detail. It is pointed out that (1) unlike the case of near circular orbits, the J_{32} term does not dominate the 2:1 mean motion resonance problem (intermediate period motion), and that (2) instead of the J_2^2 terms, the resonant tesseral harmonics dominate the critical inclination problem (long period motion).

Keywords: Molniya orbit; Mean motion resonance; Critical inclination; Double resonance; Large eccentricity

1. Introduction

On the basis of the glorious products of the orbit theories, various special artificial satellite orbits are developed for certain missions, one of which is the Molniya orbit. It is a type of highly elliptical orbit with an inclination of 63.4°, an argument of perigee of 270° and an orbital period of one half of a sidereal day. The dynamics of a Molniya orbit is complicated due to the combined effects of the critical inclination and the 2:1 mean motion resonance. The large eccentricity, usually over 0.7, adds some difficulties to the problem.

The mean motion resonance and the critical inclination problem have been intensively studied since 1960s. The principal perturbations on the near circular geosynchronous and semi-synchronous satellites orbiting the Earth arise from the J_{22} and J_{32} tesseral harmonics, respectively (Blitzer et al., 1962; Blitzer, 1965; Gedeon, 1969; Nacozy and Diehl, 1982; Sochilina, 1982; Sampaio et al., 2012; Zhao et al., 2013; Zhang et al., 2013). The well known Von Zeipel correction terms, proportional to J_2^2 in the averaged Hamiltonian, dominate the classical critical inclination problem (Hori, 1960; Jupp, 1975, 1980, 1987). The *Ideal Resonance Model*, firstly presented by Garfinkel, is

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applied to study both problems (Garfinkel, 1966; Deprit, 1969; Jupp, 1972, 1973; Garfinkel and Williams, 1974; Garfinkel, 1975). Using this model, Liu et al. (1991) derives the libration width and the centre period, considering the J_{22} (J_{32}) tesseral terms for the 24-h (12-h) orbits, and the J_2^2 terms in the classical critical inclination problem. They present that the libration width of i is $\Delta i \approx 0.04^\circ$ for the near circular, critically inclined low Earth orbits. This width is so small that the phase structure of the resonance is easy to disrupt under the effects of other perturbations.

However, the combined effects of the critical inclination and the mean motion resonance have rarely been investigated. This problem can be reduced to a double resonance model. Henrard (1990) presents a semi-numerical perturbation method for separable two-degree-of-freedom Hamiltonian systems, and his main idea is that to numerically integrate the motion of the separated degree-of-freedom and then to obtain a first integral, independent of the Hamiltonian. Delhaise and Henrard (1993) reduce the combined resonance problem to a near separable double resonance model, in view of the existence of two kinds of oscillations with one's period approximately 100 times larger than the other's. Hence, they apply the semi-numerical method to this problem to obtain the global secular dynamics.

Henrard's semi-numerical method is generally applicable to the most separable or near separable two-degree-of-freedom Hamiltonian systems. Instead, analytic methods are discussed and applied in this paper to analyse the long term evolution of the Molniya orbits. That is reasonable because the intermediate period motion of such orbits can be well approximated by the pendulum model. A detailed description of the study logic is provided in the following.

In Section 2, a simplified double resonance model (Eq. (10)) is established to describe the long term evolution of the Molniya orbits. Equilibria and their stabilities are studied, and the results are listed in Table 1. The double resonance model is separated into two one-degree-of-freedom Hamiltonian systems (Eq. (16)), according to the magnitude of the coefficients in Eq. (10).

In Section 3, the unperturbed part of the Hamiltonian system, determined by $\mathcal{H}_0$, is solved. The solution listed in Eq. (19)–(22) is a good approximation to the intermediate period motion of the Molniya orbits.

In Section 4, the long period motion of the Molniya orbits is studied qualitatively. The long term perigee evolution of the Molniya orbits are compared with that of the critically inclined orbits without mean motion resonance. The J_2^2 terms dominate the long term perigee motion if the orbit is critically inclined and is not in the state of mean motion resonance. In the combined resonance problem (i.e. the Molniya orbits), the resonant tesseral terms dominate, since they cannot be removed as short periodic terms. The libration widths of the two problems, presented in Eqs. (27) and (30), are compared to show that the mean motion resonance enhances the perigee resonance (i.e. the so-called critical inclination problem).

In Section 5, the phase structures of the long period motion are studied, based on the analytic expression of a first integral $\bar{I}_3$. Depending on the fact that the intermediate period motion librates or circulates, three cases (A, B and C) are discussed separately. Three basic types (I, II, III), depending on the equilibria and their stabilities, of the phase structures are discovered by choosing different values of the Hamiltonian constant h. All of these three types are observed in case A and only one type (II) is observed in cases B and C. Anomalies are detected in case C when the intermediate period motion is near the separatrix between libration and circulation. Numerical tests are applied to verify the phase structures of the long period motion at the end of the Section 5.

Table 1
Equilibria and their stabilities.*

$x_2(^\circ)$	$x_3(^\circ)$	stability
0.7604	70.25	unstable
86.743	74.34	stable
173.64	78.06	unstable
266.33	75.65	stable

* Extremal points of $\mathcal{F}_{2T}^c$ as a function of x_2 and x_3 are listed. Here the local maximum points correspond to the stable equilibria. The local minimum points correspond to the unstable equilibria. The Keplerian elements a, e, i in the coefficients $B_{lmpq}(a, e, i)$ are fixed at the critical values (see Eq. (12)).

2. Double resonance model

2.1. Construction of the model

Since a Molniya orbit is in the state of the 2:1 mean motion resonance, we introduce the following canonical variables (Giacaglia, 1969; Delhaise and Henrard, 1993):

$$\begin{aligned} &x_1 = \ell, \quad x_2 = g, \quad x_3 = \frac{1}{2}\ell + h \\ &y_1 = L - \frac{1}{2}H, \quad y_2 = G, \quad y_3 = H \end{aligned} \tag{1}$$

with L, G, H, ℓ, g, h being the Delaunay variables:

$$\begin{aligned} &L = \sqrt{\mu a}, \quad G = L\sqrt{1-e^2}, \quad H = G\cos i \\ &\ell = M, \quad g = \omega, \quad h = \Omega - S_g \end{aligned} \tag{2}$$

where $a, e, i, M, \omega, \Omega$ are the keplerian elements and S_g is the sidereal time.

The Hamiltonian for a Molniya orbit is averaged over the short period motion, considering the Earth's J_2 harmonic and the tesseral harmonics. Up to the second order of the J_2 harmonics, it takes the following form (Delhaise and Henrard, 1993):

$$\mathcal{F} = \mathcal{F}_0 + \mathcal{F}_1 + \mathcal{F}_{2T} + \mathcal{F}_{2Z} \tag{3}$$

with

$$\left\{ \begin{aligned} &\mathcal{F}_0 = -\frac{\mu^2}{2L^2} - \omega_e H \\ &\mathcal{F}_1 = -\frac{\mu^4}{L^6}\frac{R_e^2 J_2}{4}\left(3\frac{H^2}{G^2} - 1\right)\left(\frac{L}{G}\right)^3 \\ &\mathcal{F}_{2T} = -\sum_{m=2(l-2p+q)} B_{lmpq}(a,e,i) S_{lmp}(x_2, x_3) \\ &\mathcal{F}_{2Z} = -\frac{\mu^6 R_e^4 J_2^2}{4L^{10}}\left[A(L,G,H)\cos 2x_2 + C(L,G,H)\right] \end{aligned} \right. \tag{4}$$