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Regional gravity field model in Pakistan area from the combination of CHAMP, GRACE and ground data using least squares collocation: A case study

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Abstract

This study describes a methodology of recovery of the Earth's gravity field from CHAMP and GRACE satellites data in Pakistan using least squares collocation (LSC) based downward continuation technique. The CHAMP height anomalies and GRACE gravity disturbances derived from the observed satellite data have been used in combination solution using LSC with observed gravity values at the Earth surface. The combined covariance functions of height anomalies and/or gravity disturbances at satellite altitudes and observed gravity anomalies at Earth surface have been used as the basis for combination and downward continuation solution. The variance of predicted gravity anomalies from GRACE gravity disturbances is relatively lower than the corresponding results of gravity anomalies from CHAMP height anomalies. This fact may be attributed partly to the amplification of noise and partly to the unstable inverse transformation process of height anomalies to gravity anomalies. The impact of data error variance has been studied in the context of smoothing and noise reduction in the final solution of downward continuation using least squares collocation. The raising of data error suppresses the noise and as a result a smooth final solution is obtained. The prediction results appear to be dependent on the quality of data and goodness of combined covariance function, which are fairly comparable for the CHAMP and GRACE data. The recovered gravity field from satellite data appears to contribute mainly to medium and long wavelength parts of total gravity field spectrum. Due to flexibility of data handling in least squares collocation, this procedure is applicable to any observable of gravity field being at different altitudes and with different data spacing.

Keywords: Downward continuation; Smoothing; Covariance function; Gravity field recovery; Error variance; Regularization

1. Introduction

The accurate determination of the Earth's gravity field surely benefits almost all physical sciences especially in the field of geophysics and geodesy. Better knowledge of Earth's gravity field facilitates determination of better geoid and more precise satellite positioning and orbit determination in space. The satellite methods of gravity field determination have become an important alternative to terrestrial and airborne gravimetric methods. Due to capability of global coverage, Satellite-to-Satellite Tracking (SST) could be used for mapping those areas of the world for which limited and/or no gravimetric observation is available.

Even though they provide solutions with less resolution than terrestrial and airborne gravimetric methods, they can significantly contribute to precise determination and improvement of the knowledge of medium and long wavelengths of gravity field and their temporal variations. There are two important SST configurations i.e. there are two satellite configurations comprising High–Low (HL) and

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Low–Low (LL), out of which the Low–Low type appears to map the Earth's gravity field with better resolution (Reigber, 1988). Furthermore, LL–SST configuration can be combined with the High–Low concept, e.g. in the GRACE mission to provide much higher accuracy.

Due to the mission profile and the high accuracy achievable by the dedicated instrumentation, a further quantum leap in the accuracy of the Earth's gravity model is seen with GRACE as compared to CHAMP (Reigber, 1988). In this case, the two orbiting satellites with microwave link are strongly perturbed by short wavelength variations of the gravity field. Since the two GRACE satellites are separated in orbit by only few hundred kilometres (~220 km) along-track, the errors of the measured inter-satellite range due to environmental effects are smaller as compared to space or ground-based tracking. The non-gravitational forces caused by drag or solar radiation pressure are to be estimated using high accuracy accelerometers mounted onboard on each satellite. The orbit positions are continuously and accurately determined by geodetic space-qualified GPS receivers, which can be externally calibrated by dedicated Laser Retro reflectors integrated on each satellite. The orbits of the two satellites, which are dependent on the integrated effect of the mass distributions and movements in the Earth's system, are influenced by these effects at slightly different phases and will therefore be perturbed slightly differently. This perturbation difference will cause changes in the inter-satellite range, which is measured very precisely (<1 μm/s) by the K-band ranging equipment. These differential measurements can be used to monitor higher frequency contents of the gravitational signal to improve higher resolution estimates of the Earth's gravity field.

There are mainly four approaches used for the recovery of gravity field from SST observables. One approach is to solve for the spherical harmonic coefficients of the global geopotential models as proposed by Sjoberg (1982), Wagner (1983), Reigber (1988) and Tapley (1973). Second approach is to process GRACE data using the nonlinear orbit determination and parameters recovery method (Tapley, 1973). Third approach to obtain gravity field parameters consists of estimating the gravity induced linear orbit perturbations. This method is based on the fact that among all perturbing forces acting on the satellite, the force produced by the anomalous part of the geopotential appears to be dominating especially for the low orbiting satellites. Moreover, this method uses the classical decomposition of the disturbing potential into Fourier components of ordinary Kepler elements (Kaula, 1966). Fourth approach is the energy integral scheme based on energy conservation principle (Jekeli, 1999) used on the satellite body that is widely used to process CHAMP and GRACE data. One important alternative to the above methods is the direct use of observations as values on a boundary surface. These values can be related to the geopotential gravitational field either through spherical harmonic expansion, suitable for global model solutions, or integral inversion like Poisson's integral or least squares collocation for regional gravity field determination. The application and feasibility of the last method based on least squares collocation has been evaluated in the present study. For practical application, CHAMP satellite height anomalies and GRACE gravity disturbances have been used as individual data as well as in a combination solution with terrestrial gravity observations in the downward continuation. Additionally, the effect of data error variance (or standard deviation) on noise suppression and regularization for the use LSC technique has been studied. The paper has been arranged in the following sequence. Section 2 describes the procedure of downward continuation with least squares collocation, Section 3 includes details on processing of satellite and ground data, Section 4 provides the analysis of results and Section 5 concludes the study with some conclusions and recommendations.

2. Downward continuation with least squares collocation

The downward continuation of gravity data is an ill-posed problem due to the reason that gravity field attenuates with altitude. It is a high-pass filtering process and causes small measurements and systematic errors to amplify along with data. Therefore downward continuation of the SST measurement needs to be stabilized by an appropriate regularization or smoothing procedures. A careful study of the data and model errors is required for stable results from downward continuation. There are two main techniques used for the evaluation of downward continuation problem. The first uses the Poisson's integral formula which is purely deterministic approach. Various implementations of the method have been presented during past years, including pure integral solutions, e.g. Nov'ak et al. (2001a, 2001b) and Kern (2003), as well as their spectral equivalents, e.g. Forsberg (1987). The second method is based on the least squares collocation theory as described by Moritz, (1980). Different authors have addressed the downward continuation problem with different regularization techniques. The examples include least squares collocation (LSC) with smoothed data (Hajela, 1977, 1978, 1979), stabilized integrals (Rummel, 1980), singular value decomposition (Schwarz and Gerstl, 1979), the Tikhonove method (Tikhonov and Arsenin, 1974; Ilk, 1993; Ilk et al., 2002) and the conjugate gradient methods (Schuh et al., 1996). The main issue in this case is the determination of an optimum regularization, which can be achieved by the use of proper data reductions and filtering (Olesen et al., 2001). The satellite data especially for the cases of CHAMP and GRACE satellites has a coloured noise over the whole frequency spectrum with strong time correlation and can be isolated through filtering. A fundamental disadvantage of the satellite derived gravity data is the attenuation of high frequencies due to the flight altitude. For this reason the downward continuation procedure is introduced with special treatments to stabilize the solution and to avoid the amplification of short wavelength noise. In practical