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The magnetosphere as a complex system

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Abstract

The magnetosphere is a multi-scale spatio-temporal complex dynamical system. In this context, we have analyzed the multifractal behavior of the AL index, as a proxy for an energy dissipation rate, using discrete wavelet leaders. This technique allows the calculation of the spectrum for both positive and negative values of q, giving a robust peak at $h \approx 0.5$. The same technique is applied to the dissipation rate of a simple 1D model of intermittent magnetic field annihilation, showing a clear multifractal behavior, but with a peak at $h \approx 0.2$. Even though this intuitive 1D model, because of its simplicity, is not expected to reproduce all the complex dynamics that occur in the Earth's magnetotail, it suggests that the existence of a multifractal dissipation dynamics is necessary to the establishment of the self-organized state, as shown in a 2D simulation of intermittent plasma dynamics.

Keywords: Self-organizations; Space plasmas; Substorms; Multifractals

1. Introduction

There is mounting evidence that plasmas can display very complex behavior, that includes multi-scale dynamics, emergence and self-organization, phase transitions, turbulence, spatio-temporal chaos, etc. (Lu, 1995; Carreras et al., 1996; Biskamp, 2000) The study of self-organization (SO) and its relation with turbulence is a subject at the forefront of astrophysics and space research (Chang, 1999), and in particular, it may have relevance in the behavior of the magnetospheric dynamics.

In this respect, the magnetosphere is a multi-scale spatio-temporal complex dynamical system. The concept of self-organization may contribute to resolve two seemingly contradicting observations (Chang, 1999): (a) the magnetotail plasma sheet appears to be a dynamic and self-similar turbulent region (Borovsky et al., 1997; Ohtani et al., 1998; Pinto et al., 2011a,b), and (b) the substorm cycle seems coherent and repeatable with identifiable distinct phases (Baker et al., 1999) and predictable geomagnetic indices (Vassiliadis et al., 1995; Valdivia et al., 1996, 1999). We suggest that these seemingly contradicting statements may be reconciled by proposing that the plasma sheet is driven into a non-equilibrium self-organized "global" state (Chang, 1999), as suggested initially by Chang (1992), that is characterized by critical behavior with scale invariant events, self-similar spatial structure, and multifractal topology. Such states, are seen to emerge naturally in plasma physics models with sporadic dissipation, through spatio-temporal chaos (Klimas et al., 2000, 2004; Valdivia et al., 2005; Chang, 1992; Ugai and Tsuda, 1977). This paradigm is in sharp contrast to the standard picture of plasma sheet transport with laminar earthward flow in a well ordered magnetic field. Instead they are more consistent with the presence of elementary transport events, probably bursty bulk flows (Baumjohann et al., 1990; Angelopoulos et al., 1992), that are accelerated in local reconnection regions (see schematic diagram of Fig. 1a).

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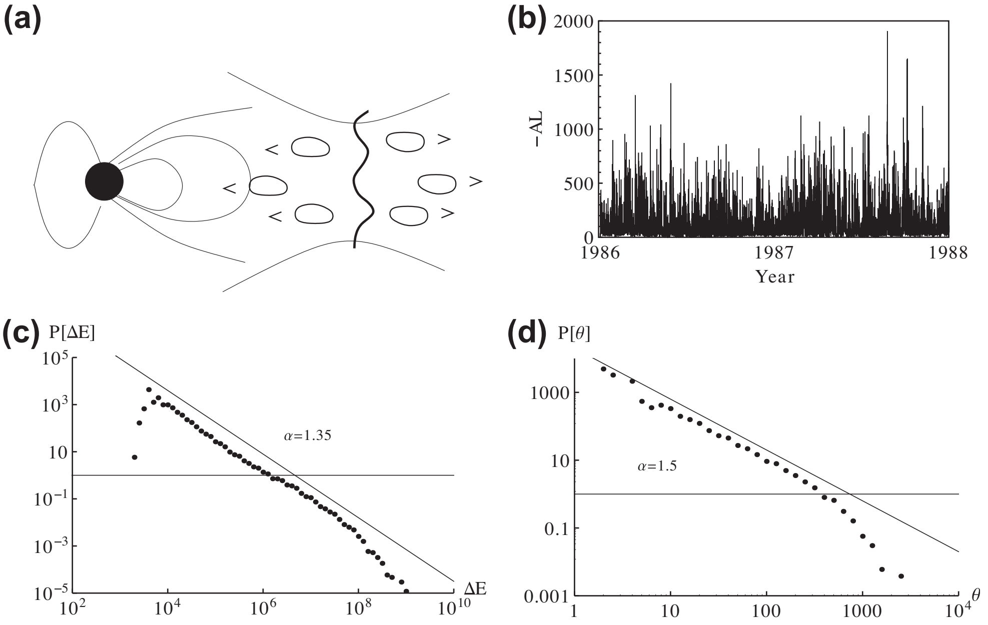

Fig. 1. (a) Conceptual view of the complex magnetosphere. (b) -AL time series. (c) The energy release distribution ΔE, and the (d) waiting time distribution for the dissipation proxy AL^2.

We will study some issues relating to the complex behavior of the magnetosphere, and relate them to the multifractal intermittent energy dissipation in simple plasma models of a current sheet. In particular, we will analyze how the underlying localized turbulence can self-organize into a global state under different driving conditions. The multi-scale behavior present in this model seems to occur naturally in complex systems, and is of particular relevance for the existence of an out-of-equilibrium globally stable state with underlying multifractal turbulent behavior. The complex behavior of this system will also be studied using the techniques that are being developed for spatio-temporal chaotic dynamics.

There is mounting evidence that such a SO state occurs in the magnetosphere. Consolini (1997) found a power law distribution of burst strength in the AL index. For the 2 years of the AL index shown in Fig. 1b let's use AL^2 as a rough proxy for the energy dissipation rate. Obviously this is not correct, for we don't have the conductivity nor the effective area of dissipation. Still, we computed the event distribution P of the energy dissipated ΔE, when $AL^2 > (50\ \mathrm{nT})^2$. If we assume $P(\Delta E|\alpha) = \Delta E^{-\alpha}/\zeta(\alpha)$, with $\zeta(\alpha)$ as the Riemann zeta function, and apply a Bayesian argument to the measured sequence, we estimate the power law index $\alpha_{\Delta E} \sim 1.35$ from the maximum of (Goldstein et al., 2004)

$$P(\alpha|\{\Delta E\}) \sim \Pi_i P(\Delta E_i|\alpha) = e^{-\alpha \sum_i^N \ln(\Delta E_i) - N\zeta(\alpha)},$$

independently of the binning process (we assume a smooth prior $P(\alpha)$). N is the number of data points. The waiting time between events is also shown in Fig. 1d displaying a power law distribution with index $\alpha_\theta \sim 1.5$. This power law distribution in waiting times is of fundamental nature, and can be used to discriminate between models. For example, a simple sandpile model will have problems in generating such a waiting time distribution (Boffetta et al., 1999).

We know that the magnetosphere, and in general turbulence (Terry, 2000), could be represented by a complex set of very nontrivial equations, however, many approaches take the complementary route and focus on the self-similarity associated with this behavior to illustrate the basic nature of the dynamics (Chang, 1999; Frisch, 1996). We will see that the multifractal dissipation observed in the magnetospheric dynamics is a robust characteristic and fundamental to the understanding and modeling of its evolution. In turn, this behavior can be used to discriminate the models that can be used to describe its evolution.

Traditionally, a nonlinear variation of the exponent ξ_q with respect to q, in the structure function

$$\left\langle \left| AL^2(t+\tau) - AL^2(t) \right|^q \right\rangle \sim \tau^{\xi_q}, \tag{1}$$

has been used as an indication of the intermittent multifractal dissipation in this spatio-temporal system (Valdivia et al., 2006), where $\langle\rangle$ corresponds to the average over all data points. Estimating ξ_q directly from the above structure function suffers from a number of drawbacks, such as the effect of noise for large values of q (specially negative values), simplicity of the structure function that makes it prone to noise effects, persistent effects in the time series that need to be detrended, among other issues. We will now describe a more robust method to study the complex behavior using discrete wavelet transforms (Frisch, 1996; Jaffard et al., 2001; Muzy et al., 1991, 1993; Lashermes et al., 2008), which will show that the multifractal behavior in the magnetosphere is robust, and cannot be avoided in any description of the system.