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Two-manoeuvres transfers between LEOs and Lissajous orbits in the Earth–Moon system

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Abstract

The purpose of this work is to compute transfer trajectories from a given Low Earth Orbit (LEO) to a nominal Lissajous quasi-periodic orbit either around the point L_1 or the point L_2 in the Earth–Moon system. This is achieved by adopting the Circular Restricted Three-Body Problem (CR3BP) as force model and applying the tools of Dynamical Systems Theory.

It is known that the CR3BP admits five equilibrium points, also called Lagrangian points, and a first integral of motion, the Jacobi integral. In the neighbourhood of the equilibrium points L_1 and L_2, there exist periodic and quasi-periodic orbits and hyperbolic invariant manifolds which emanate from them. In this work, we focus on quasi-periodic Lissajous orbits and on the corresponding stable invariant manifolds.

The transfers under study are established on two manoeuvres: the first one is required to leave the LEO, the second one to get either into the Lissajous orbit or into its associated stable manifold. We exploit order 25 Lindstedt–Poincaré series expansions to compute invariant objects, classical manoeuvres and differential correction procedures to build the whole transfer.

If part of the trajectory lays on the stable manifold, it turns out that the transfer's total cost, Δv_{tot}, and time, t_{tot}, depend mainly on:

1. the altitude of the LEO;
2. the geometry of the arrival orbit;
3. the point of insertion into the stable manifold;
4. the angle between the velocity of insertion on the manifold and the velocity on it.

As example, for LEOs 360 km high and Lissajous orbits of about 6000 km wide, we obtain $\Delta v_{\text{tot}} \in [3.68, 4.42]$ km/s and $t_{\text{tot}} \in [5, 40]$ days.

As further finding, when the amplitude of the target orbit is large enough, there exist points for which it is more convenient to transfer from the LEO directly to the Lissajous orbit, that is, without inserting into its stable invariant manifold.

Keywords: Earth–Moon transfers; Circular Restricted Three-Body Problem; Lissajous orbits

1. Introduction

Forty years after the first step by a man on the Moon, we are witnesses of a debate, active more than ever, on a possible return. Not just NASA, but also India, China and Japan have designed unmanned missions that are now orbiting the Moon looking for water, testing new technology and obtaining a detailed characterization of the satellite in view of a future human installation. Also, the space tourism companies are planning to extend their potentiality by offering trips to the Moon.

In this framework, we apply the model of the Circular Restricted Three-Body Problem (CR3BP) to the Earth–Moon system and we look for trajectories going from a

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nominal orbit around the Earth to a nominal quasi-periodic orbit around the collinear equilibrium point either L_1 or L_2. As L_1 is located between the two primaries, its neighbourhood seems to be the most appropriate place to put a space hub, serving for instance as a construction and repair facility. On the other hand, L_2 would be profitable to monitor the lunar farside. The transfers we investigate would be also useful if considering other types of Solar System explorations, for instance to go to Mars or to a minor body.

It is well-known that the CR3BP, in a proper reference system, admits five equilibrium points (Szebehely, 1967) and that central and hyperbolic invariant manifolds originate from the neighbourhood of the collinear ones. Among the orbits filling the central invariant manifold, we focus on the Lissajous ones, quasi-periodic orbits lying on invariant tori. This two-parameter family of solutions imposes less constraints to the mission designer than the widely used halo orbits, essentially because the in-plane and the out-of-plane amplitudes can be chosen independently one to the other. As further advantage, the eclipse avoidance problem can be solved in a non-expensive way. Also, a methodology for transfers involving Lissajous orbits can be extended and applied to the periodic orbit case.

As a matter of fact, in the Earth–Moon system the hyperbolic manifolds associated with central orbits pass quite far from our planet. The approach we follow to achieve the transfer is based on two manoeuvres: one to depart from the Low Earth Orbit (LEO) and one to insert either into the Lissajous or into one of the branches of its stable invariant manifold. We intend to provide a global picture of the dynamics driving the transfers, with special emphasis on the role played by the geometry of the arrival orbits and of the stable manifolds. We will demonstrate that the distance existing between the LEO and the points on the manifold is crucial for a cheap connection.

Recently, other authors have considered the same problem with different methodologies. Parker (2007) and Rausch (2005) fixed as arrival locations halo orbits around the point L_1 in the Earth–Moon system. The first author computed a 2-manoeuvres connection by means of a Two-Body Problem (2BP) approximation refined including the gravitational effect of the Moon. Rausch used a shooting technique to construct a continuous arc linking two given points in a fixed time of flight. Renk and Hechler (2008) exploited optimization techniques in order to transfer from a nominal LEO to a nominal Libration Point Orbit (LPO) (halo and Lissajous) either around L_1 or L_2. The trajectories computed follow the escape directions associated with the LPO and may perform a lunar fly-by. Gordon (2008) focused his work on LEO–LPO around the point L_2. In his approach, a differential correction procedure is used to meet some constraints at the departure and at the insertion either into a planar Lyapunov orbit or into a halo orbit.

In this paper, after a short description of the model, we explain the semi-analytical and numerical tools employed for the computation of the invariant objects and for the LEO-manifold trajectory. After that, we present the results obtained, trying to point out the differences between our strategy and the above-mentioned studies.

2. The model

The Circular Restricted Three-Body Problem (Szebehely, 1967) studies the behavior of a particle P with infinitesimal mass m_3 moving under the gravitational attraction of two primaries P_1 and P_2, of masses m_1 and m_2, revolving around their center of mass in circular orbits.

To remove time from the equations of motion, it is convenient to introduce a synodical reference system $\{O, x, y, z\}$, which rotates around the z-axis with a constant angular velocity ω equal to the mean motion n of the primaries. The origin of the reference frame is set at the barycenter of the system and the x-axis on the line which joins the primaries, oriented in the direction of the largest primary. In this way, we work with m_1 and m_2 fixed on the x-axis, as shown in Fig. 1.

The units are chosen in such a way that the distance between the primaries and the modulus of the angular velocity of the rotating frame are unitary. This means that, for the Earth–Moon system, the unit of distance equals 384,400 km, the unit of velocity equals 1.02316 km/s and the dimensionless mass of the Moon is $\mu = 0.012150582$. With these assumptions, the equations of motion can be written as

$$
\begin{aligned}
\ddot{x} - 2\dot{y} &= x - \frac{(1-\mu)}{r_1^3}(x-\mu) - \frac{\mu}{r_2^3}(x+1-\mu),\\
\ddot{y} + 2\dot{x} &= y - \frac{(1-\mu)}{r_1^3}y - \frac{\mu}{r_2^3}y, \qquad (1)\\
\ddot{z} &= -\frac{(1-\mu)}{r_1^3}z - \frac{\mu}{r_2^3}z,
\end{aligned}
$$

where $r_1 = [(x-\mu)^2 + y^2 + z^2]^{\frac{1}{2}}$ and $r_2 = [(x+1-\mu)^2 + y^2 + z^2]^{\frac{1}{2}}$ are the distances from P to P_1 and P_2, respectively.

The system (1) has a first integral, the *Jacobi integral*, which is given by

$$
C = x^2 + y^2 + \frac{2(1-\mu)}{r_1} + \frac{2\mu}{r_2} + (1-\mu)\mu - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2). \qquad (2)
$$

In the synodical reference system, there exist five equilibrium (or *libration*) points (see Fig. 1). Three of them, the *collinear* ones, are in the line joining the primaries and are usually denoted by L_1, L_2 and L_3. If x_{L_i} $(i = 1, 2, 3)$ denotes the abscissa of the three collinear points, we assume that

$$x_{L_2} < \mu - 1 < x_{L_1} < \mu < x_{L_3}.$$

The collinear libration points behave as the product of two centers by a saddle. When we consider all the energy levels, the center $\times$ center part gives rise to four-dimen-