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Magnetogasdynamic shock wave generated by a moving piston in a rotational axisymmetric isothermal flow of perfect gas with variable density

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Abstract

The propagation of a strong cylindrical shock wave in an ideal gas with azimuthal magnetic field, and with or without axisymmetric rotational effects, is investigated. The shock wave is driven out by a piston moving with time according to power law. The ambient medium is assumed to have radial, axial and azimuthal component of fluid velocities. The fluid velocities, the initial density and the initial magnetic field of the ambient medium are assumed to be varying and obey power laws. Solutions are obtained, when the flow between the shock and the piston is isothermal. The gas is assumed to have infinite electrical conductivity and the angular velocity of the ambient medium is assumed to be decreasing as the distance from the axis increases. It is expected that such an angular velocity may occur in the atmospheres of rotating planets and stars. The shock wave moves with variable velocity and the total energy of the wave is non-constant. The effects of variation of the initial density and the Alfven-Mach number on the flow-field are obtained. A comparison is also made between rotating and non-rotating cases.

Keywords: Shock wave; Stars: rotation; Magnetogasdynamics; Interplanetary medium

1. Introduction

The experimental studies and astrophysical observations show that the outer atmosphere of the planets rotates due to rotation of the planets. Macroscopic motion with supersonic speed occurs in an interplanetary atmosphere and shock waves are generated. Thus the rotation of planets or stars significantly affects the process taking place in their outer layers, therefore question connected with the explosions in rotating gas atmospheres are of definite astrophysical interest. Chaturani (1970) studied the propagation of cylindrical shock wave through a gas having solid body rotation, and obtained the solutions by a similarity method adopted by Sakurai (1956). Nath et al. (1999) obtained the similarity solutions for the flow behind the spherical shock waves propagating in a non-uniform rotating interplanetary atmosphere with increasing energy. Ganguly and Jana (1998) studied a theoretical modal of propagation of strong spherical shock waves in a self-gravitating atmosphere with radiation flux in presence of a magnetic field. They also, considered the medium behind the shock to be rotating, but neglected the rotation of the undisturbed medium. Vishwakarma et al. (2007) obtained the similarity solution for self-similar adiabatic flow headed by a magnetogasdynamic cylindrical shock wave in a rotating non-ideal gas.

In aerodynamics the analogy between the steady hypersonic flow past slender blunted (planar or axisymmetric) power-law bodies and the one-dimensional unsteady self-similar flow behind a shock driven out by a piston moving with time according to a power-law is well known (see, for example, Grigoryan (1958), Kochina and Melnikova (1958), Wang (1964), Helliwell (1969), Sedov (1959), Rosenau and Frankenthal (1976), Steiner and Hirschler (2002), Nath (2007), Vishwakarma and Nath (2009, 2010)).

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Since at high temperatures that prevail in the problems associated with the shock waves a gas is ionized, electromagnetic effects may also be significant. A complete analysis of such a problem should therefore consist of the study of the gas-dynamic flow and the electromagnetic fields simultaneously.

In all of the works, mentioned above, the ambient medium is supposed to have only one component of velocity that is the azimuthal component.

In the present work, we obtain the self-similar solutions for the flow behind the magnetogasdynamic strong cylindrical shock wave generated by a moving piston in a rotational axisymmetric isothermal flow of a gas with variable density, which has a variable azimuthal fluid velocity together with a variable axial fluid velocity (Levin and Skopina (2004), Nath (2010)). The fluid velocities, the azimuthal magnetic field and the density in the ambient medium are assumed to vary and obey the power laws. Also, the angular velocity of rotation of the ambient medium is assumed to be obeying a power law and to be decreasing as the distance from the axis increases. It is expected that such an angular velocity may occur in the atmospheres of rotating planets and stars. The assumption of isothermal flow is physically realistic, when radiation heat transfer effects are implicitly present. As the shock propagates, the temperature behind it increases and becomes very large so that there is intense transfer of energy by radiation. This causes the temperature gradient to approach zero, that is the dependent temperature tends to becomes uniform behind the shock front and the flow becomes isothermal (Laumbach and Probstein (1970), Sachdev and Ashraf (1971), Ashraf and Ahmad (1975), Korobeinikov (1976), Zhuravskaya and Levin (1996), Nath (2007)). With this assumption, we obtain the solution in Section 3. The gas ahead of the shock is assumed to be at rest. Effects of viscosity and gravitation are not taken into account.

2. Equations of motion and boundary conditions

The fundamental equations for one-dimensional, unsteady isothermal axisymmetric rotational flow of an electrically conducting ideal gas in the presence of an azimuthal magnetic field may, in Eulerian coordinates, can be expressed as (c.f. Witham (1958), Laumbach and Probstein (1970), Sachdev and Ashraf (1971), Ashraf and Ahmad (1975), Korobeinikov (1976), Zhuravskaya and Levin (1996), Levin and Skopina (2004), Nath (2007, 2010))

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{u\rho}{r} = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\left[\frac{\partial p}{\partial r} + \mu h\frac{\partial h}{\partial r} + \frac{\mu h^2}{r}\right] - \frac{v^2}{r} = 0, \quad (2)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{uv}{r} = 0, \quad (3)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} = 0, \quad (4)$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} = 0, \quad (5)$$

$$\frac{\partial T}{\partial r} = 0, \quad (6)$$

where r and t are independent space and time coordinates; u, v, and w are the radial, azimuthal and axial components of the fluid velocity $\vec{q}$ in the cylindrical coordinates (r, θ, z); ρ, p, h and T are the density, the pressure, the azimuthal magnetic field and the temperature; μ is the magnetic permeability.

Also,

$$v = Ar, \quad (7)$$

where 'A' is the angular velocity of the medium at radial distance r from the axis of symmetry . In this case the vorticity vector

$\vec{\varsigma} = \frac{1}{2} Curl\ \vec{q}$, has the components

$$\varsigma_r = 0, \quad \varsigma_\theta = -\frac{1}{2}\frac{\partial w}{\partial r}, \quad \varsigma_z = \frac{1}{2r}\frac{\partial}{\partial r}(rv). \quad (8)$$

The vorticity vector in this flow is $\vec{\varsigma} = \varsigma_r \hat{e}_r + \varsigma_\theta \hat{e}_\theta + \varsigma_z \hat{e}_z$. To interpret this result, notice that in Figs. 1 and 2 the z-axis is pointing to the right along the axis of the cylinder (cylindrical shock). It is assumed that the fluid is moving radially, rotating and simultaneously moving down the axis of cylinder. If we focus our attention on the thin fluid element originally aligned in the radial direction with $\theta = 0$, we see that as times passes this fluid element rotates in a clockwise direction (CW) as viewed in the $+\theta$ direction owing to the non-uniform velocity profile.

The electrical conductivity of the gas is assumed to be infinite. Therefore the diffusion term from the magnetic field equation is omitted, and the electrical resistivity is

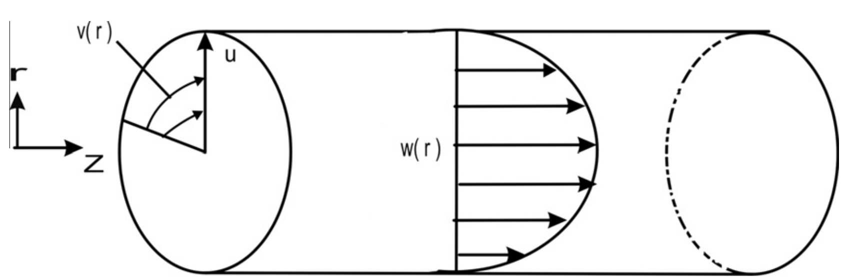

Fig. 1. The directions of velocity.