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Stability of liquid bridges subject to an eccentric rotation

V. Lapuerta *, A. Laverón-Simavilla, J. Rodríguez

E.T.S.I. Aeronáuticos (Universidad Politécnica de Madrid), Plaza de Cardenal Cisneros, 3, 28040 Madrid, Spain

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Abstract

A cylindrical liquid bridge supported between two circular-shaped disks in isorotation is considered. The effect of an offset between the rotation axis and the axis of the two supporting disks (eccentricity) on the stability of the static liquid bridge is investigated. A numerical method is used to find stable and unstable shapes and to determine the stability limit for different values of eccentricity. The calculated stability limits are compared with analytical results, recovering the same behavior. Numerical results have been also compared with the results of an experiment aboard TEXUS-23, recovering the stability limit and the equilibrium shapes.

Keywords: Liquid bridge; Microgravity; Stability

1. Introduction

The behavior of liquid bridges has been widely studied, both theoretically and experimentally, due to the use of this configuration in a crystal growth technique known as the floating zone technique (Meseguer and Sanz, 1985). In this technique, rotation of the supports is used to achieve a uniform temperature field.

In this paper a cylindrical liquid bridge supported between two circular-shaped disks in isorotation is considered. In the absence of gravity, two types of instability, namely, C-mode and amphora mode, depending on the slenderness, can appear (Vega and Perales, 1983; Perales et al., 1990). The effect of an offset between the rotation axis and the axis of the two supporting disks (eccentricity) on these stabilities is investigated.

The stability limits and the equilibrium shapes of the configuration are calculated using an extension of an already implemented numerical method (Laverón-Simavilla and Perales, 1995; Laverón-Simavilla and Checa, 1997).

The calculated stability limits are compared with the analytical results of Perales et al. (1990) (only valid for small eccentricity). The numerical method is used to find stable and unstable shapes and to determine the stability limit for different values of eccentricity, not only restricted to small eccentricity.

For the C-mode, numerical results have been also compared with the results of an experiment aboard TEXUS-23 (see Sanz et al. (1992)) recovering the stability limit and the equilibrium shapes.

2. Problem formulation

The fluid configuration consists of a liquid bridge as sketched in Fig. 1: the liquid column is held by surface tension forces between two disks of radius R_0, placed a distance L apart. Both disks are parallel and coaxial. The volume of the bridge is that corresponding to a cylindrical one: $V = \pi R_0^2 L$. The liquid and the disks are solidly rotating at an angular speed Ω around an axis which is parallel to the axis of the disks, and is placed a small distance E (eccentricity) apart from this line.

The equation governing the steady shape of the liquid bridge is obtained by expressing the equilibrium between the different forces at the interface

* Corresponding author.
E-mail address: mariavictoria.lapuerta@upm.es (V. Lapuerta).

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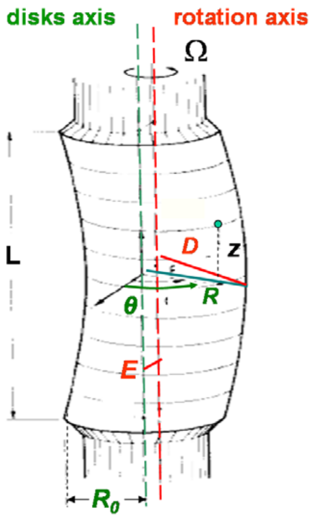

Fig. 1. Geometry and coordinate system for the liquid bridge problem.

$$\sigma\tilde{M}(R) + \tilde{P} + \frac{1}{2}\rho\Omega^2 D^2 = 0 \tag{1}$$

where $R = R(Z,\theta)$ is equation of the gas–liquid interface, σ is the surface tension, $\tilde{M}(R)$ is twice the mean curvature of the interface, $\tilde{P}$ is the pressure difference at the origin, ρ is the liquid density, Ω is the angular speed and D is the distance between a point of the free surface and the rotation axis (see Fig. 1) which, in terms of the shape of the surface and the azimuthal angle θ, can be calculated as:

$$D = (R^2 + 2ER\cos\theta + E^2)^{1/2} \tag{2}$$

Eq. (1) has to be integrated with the boundary conditions

$$R(\pm L/2,\theta) = R_0 \tag{3}$$
$$R(Z,\theta) = R(Z,\theta + 2\pi) \tag{4}$$
$$\frac{1}{2}\int_{-L/2}^{L/2} \mathrm{d}Z \int_0^{2\pi} R^2(Z,\theta)\,\mathrm{d}\theta = \pi L R_0^2 \tag{5}$$

Eq. (3) indicates that the liquid column remains anchored to the disk edges, Eq. (4) comes from the azimuthal periodicity and Eq. (5) expresses the conservation of the volume of the liquid bridge.

Let us introduce the following dimensionless variables and parameters:

$$\Lambda = L/2R_0, \quad e = E/R_0, \quad W = \rho\Omega^2 R_0^3/\sigma,$$
$$P = \tilde{P}R_0/\sigma, \quad z = Z/R_0, \quad F(z,\theta) = R(z,\theta)/R_0 \tag{6}$$

where Λ is the liquid bridge slenderness, e is the dimensionless eccentricity, W is the Weber number and P is the dimensionless reference pressure.

The formulation of the problem becomes

$$M(F) + P + \frac{1}{2}W(F^2 + 2eF\cos\theta + e^2) = 0 \tag{7}$$

with

$$M(F) = \frac{F(1+F_z^2)(F_{\theta\theta} - F) + FF_{zz}(F^2 + F_\theta^2) - 2F_\theta(F_\theta + FF_zF_{\theta\theta})}{(F^2(1+F_z^2) + F_\theta^2)^{3/2}} \tag{8}$$

The dimensionless boundary conditions for Eq. (7) are

$$F(\pm\Lambda,\theta) = 1 \tag{9}$$
$$F(z,\theta) = F(z,\theta + 2\pi) \tag{10}$$
$$\int_{-\Lambda}^{\Lambda} \mathrm{d}z \int_0^{2\pi} F^2(z,\theta)\,\mathrm{d}\theta = 4\pi\Lambda \tag{11}$$

3. Numerical method

An algorithm, based on a continuation method (Keller, 1987) capable of overpassing bifurcation points and turning points (which appear for the amphora mode and the C-mode, respectively) was developed using a finite difference method (Laverón-Simavilla and Perales, 1995) and was used to obtain the bifurcation diagrams and equilibrium shapes of liquid bridges subjected to lateral acceleration and other effects. The stable or unstable character of each of the shapes is calculated to determine the position of the stability limit.

In this paper the system of Eqs. (7)–(11) is solved by using an extension of that algorithm to liquid bridges rotating around an eccentric axis to study the effect of combined eccentricity and angular speed.

The method is based on linearizing Eqs. (7)–(11) around a known solution ($F_0(z,\theta)$, P_0) by seeking solutions of the form

$$F(z,\theta) = F_0(z,\theta) + f(z,\theta) + \mathrm{o}(|f|)$$
$$P = P_0 + p + \mathrm{o}(|p|)$$

where $|f/F_0| \ll 1$ and $|p/P_0| \ll 1$, and the character 'o' means that the terms not considered are very small compared to the smallest one retained. The leading terms obtained from Eq. (7) result in an equation for $f(z,\theta)$

$$\tilde{O}^{-3/2}\left\{\tilde{A} + \left(\tilde{B} - \frac{3\tilde{A}\tilde{Q}}{2\tilde{O}}\right)f + \left(\tilde{C} - \frac{3\tilde{A}\tilde{S}}{2\tilde{O}}\right)f_z + \left(\tilde{D} - \frac{3\tilde{A}\tilde{T}}{2\tilde{O}}\right)f_\theta\right\}$$
$$+ \tilde{E}f_{zz} + \tilde{G}f_{\theta\theta} + \tilde{H}f_{z\theta} + P_0 + p + \frac{1}{2}W(e^2 + F_0^2 + 2F_0 f)$$
$$+ eW(F_0 + f)\cos\theta + \frac{1}{2}We^2 = 0 \tag{12}$$

where $\tilde{A},\tilde{B},\tilde{C},\tilde{D},\tilde{E},\tilde{G},\tilde{H},\tilde{O},\tilde{Q},\tilde{S}$ and $\tilde{T}$ are known functions of $F_0(z,\theta)$ and P_0 and consequently of the considered point on the interface. The leading terms obtained for the boundary conditions are

$$f(z,\theta) = f(z,\theta + 2\pi) \tag{13}$$
$$F_0(\pm\Lambda,\theta) + f(\pm\Lambda,\theta) = 0 \tag{14}$$
$$\int_{-\Lambda}^{\Lambda} \mathrm{d}z \int_0^{2\pi} F_0(z,\theta)^2\,\mathrm{d}\theta + 2\int_{-\Lambda}^{\Lambda} \mathrm{d}z$$
$$\int_0^{2\pi} [F_0(z,\theta)f(z,\theta)]\,\mathrm{d}\theta = 4\pi\Lambda \tag{15}$$