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Global dynamics of high area-to-mass ratios GEO space debris by means of the MEGNO indicator

S. Valk, N. Delsate *, A. Lemaître, T. Carletti

University of Namur (FUNDP), Département de Mathématique, Unité de Systémes Dynamiques, 8, Rempart de la Vierge, B-5000 Namur, Belgium

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Abstract

In this paper we provide an extensive analysis of the global dynamics of high-area-to-mass ratios geosynchronous (GEO) space debris, applying a recent technique developed by Cincotta and Simó [Cincotta, P.M., Simó, C.Simple tools to study global dynamics in non-axisymmetric galactic potentials–I. Astron. Astrophys. (147), 205–228, 2000.], *Mean Exponential Growth factor of Nearby Orbits* (MEGNO), which provides an efficient tool to investigate both regular and chaotic components of the phase space.

We compute a stability atlas, for a large set of near-geosynchronous space debris, by numerically computing the MEGNO indicator, to provide an accurate understanding of the location of stable and unstable orbits as well as the timescale of their exponential divergence in case of chaotic motion. The results improve the analysis presented in Breiter et al. [Breiter, S., Wytrzyszczak, I., Melendo, B. Long-term predictability of orbits around the geosynchronous altitude. Advances in Space Research 35, 1313–1317, 2005] notably by considering the particular case of high-area-to-mass ratios space debris. The results indicate that chaotic orbits regions can be highly relevant, especially for very high area-to-mass ratios.

We then provide some numerical investigations and an analytical theory that lead to a detailed understanding of the resonance structures appearing in the phase space. These analyses bring to the fore a relevant class of secondary resonances on both sides of the well-known pendulum-like pattern of geostationary objects, leading to a complex dynamics.

Keywords: Solar radiation pressure; Space debris; MEGNO; Detection of chaos; Long-term evolution; Geosynchronous orbit; High area-to-mass ratios; Secondary resonances

1. Introduction

Recent optical surveys in high-altitude orbits, performed by the European Space Agency 1 m telescope on Tenerife (Canary islands), have discovered a new unexpected population of 10 cm sized space debris in near geosynchronous orbits (GEO). These objects sometimes present highly eccentric orbits with eccentricities as high as 0.55 (Schildknecht et al., 2004; Schildknecht et al., 2005). Following the initial guess of Liou and Weaver (2004) who suggested that this new population may be constituted by GEO objects with high area-to-mass ratios, recent numerical and analytical investigations were performed to support this assumption (Anselmo and Pardini, 2005; Liou and Weaver, 2005). In addition, these authors and others, such as Chao (2006) and later Valk et al. (2008), presented some detailed results concerning the short- and long-term evolution of high area-to-mass ratios geosynchronous space debris subjected to direct solar radiation pressure. More specifically, these latter authors mainly focused their attention on the long-term variation of both the eccentricity and the inclination vector. Moreover, some studies concerning the effects of the Earth's shadowing effects on the motion of such space debris were given in Valk and Lemaître (2008). However, nobody ever dealt with the question to know whether these orbits

* Corresponding author.
E-mail address: nicolas.delsate@ fundp.ac.be (N. Delsate).

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are really predictable or not on the time scales of their investigations.

The objective of this paper is twofold. The first goal is the investigation of the long-term stability of high area-to-mass ratio space debris subjected to direct solar radiation pressure, by means of the *Mean Exponential Growth factor of Nearby Orbits* (MEGNO) criterion. Second, still considering high area-to-mass ratios, we bring to the fore a relevant class of additional secondary structures appearing in the phase space.

The paper is organized as follows. In Section 2, we focus our attention to the specification of the underlying model and we give some details about the numerical aspects of the method. In Section 3, for the sake of completeness, we dwell upon the detailed definition of the MEGNO indicator, also providing a review of its main properties, in order to understand the behavior of the chaos indicator. Then in Section 4, in the framework of the validation of our implementation, we retrieve the results obtained by Breiter et al. (2005). We also discuss the significance of the time of integration, recently reported by Barrio et al. (in press). In Section 5, we apply the MEGNO technique in order to give a insightful understanding of the stability of high area-to-mass ratio space debris. More specifically, we show that the orbits of such peculiar space debris are extremely sensitive to initial conditions, especially with respect to the mean longitude and the semi-major axis. Second, we perform extended numerical analyses, showing that the related 2-dimensional phase space is dominated by chaotic regions, in particular when the area-to-mass ratio is large. In addition, we also provide some results presenting the importance of the initial eccentricity value in the appearance of chaotic regions. Finally, in Section 6, we present extensive numerical and analytical investigations of the additional patterns which will be identified as secondary resonances.

2. The model

For the purpose of our study, we consider the modeling of a space debris subjected to the influence of the Earth's gravity field, to both the gravitational perturbations of the Sun and the Moon as well as to the direct solar radiation pressure. As a consequence the differential system of equations governing the dynamics is given by

$$\ddot{\mathbf{r}} = \mathbf{a}_{\text{pot}} + \mathbf{a}_{☾} + \mathbf{a}_{\odot} + \mathbf{a}_{\text{rp}},$$

where $\mathbf{a}_{\text{pot}}$ is the acceleration induced by the Earth's gravity field, which can be expressed as the gradient of the following potential

$$U(r,\lambda,\phi) = -\frac{\mu}{r}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{R_e}{r}\right)^n \mathscr{P}_n^m(\sin\phi)(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda), \tag{1}$$

where the quantities C_{nm} and S_{nm} are the spherical harmonics coefficients of the geopotential. The Earth's gravity field adopted is the EGM96 model (Lemoine et al., 1987). In Eq. (1), μ is the gravitational constant of the Earth, R_e is the Earth's equatorial radius and the quantities (r,λ,ϕ) are the geocentric spherical coordinates of the space debris. $\mathscr{P}_n^m$ are the well-known Legendre functions. It is worth noting that the potential of Eq. (1) is subsequently expressed in Cartesian coordinates by means of the Cunningham algorithm (Cunningham, 1970).

Both the accelerations $\mathbf{a}_{☾}$ and $\mathbf{a}_{\odot}$ result from the gravity interaction with a third body of mass m_*, where $* = ☾$ and $* = \odot$, and can be expressed with respect to the Earth's center of mass as

$$\mathbf{a}_* = -\mu_*\left(\frac{\mathbf{r}-\mathbf{r}_*}{\|\mathbf{r}-\mathbf{r}_*\|^3} + \frac{\mathbf{r}_*}{\|\mathbf{r}_*\|^3}\right),$$

where $\mathbf{r}$ and $\mathbf{r}_*$ are the geocentric coordinates of the space debris and of the mass m_*, respectively. The quantity μ_* is the gravitational constant of the third-body. In our implementation, we chose the high accurate solar system ephemeris given by the Jet Propulsion Laboratory (JPL) to provide the positions of both the Sun and the Moon (Standish, 1998).

Regarding direct solar radiation pressure, we assume an hypothetically spherical space debris. The albedo of the Earth is ignored and the Earth's shadowing effects are not taken into account either. The acceleration induced by direct solar radiation pressure is given by

$$\mathbf{a}_{rp} = C_r P_r \left[\frac{a_{\odot}}{\|\mathbf{r}-\mathbf{r}_{\odot}\|}\right]^2 \frac{A}{m}\frac{\mathbf{r}-\mathbf{r}_{\odot}}{\|\mathbf{r}-\mathbf{r}_{\odot}\|},$$

where C_r is the adimensional reflectivity coefficient (fixed to 1 further on in this paper) which depends on the optical properties of the space debris surface; $P_r = 4.56\times 10^{-6}\ \text{N/m}^2$ is the radiation pressure for an object located at the distance of 1 AU; $a_{\odot} = 1$ AU is a constant parameter equal to the mean distance between the Sun and the Earth and $\mathbf{r}_{\odot}$ is the geocentric position of the Sun. Finally, the coefficient A/m is the so-called area-to-mass ratio where A and m are the effective cross-section and mass of the space debris, respectively.

3. The mean exponential growth factor of nearby orbits

For the sake of clarity we present in this section the definition and some properties of the MEGNO criterion.

Let $\mathscr{H}(\mathbf{p},\mathbf{q})$, with $\mathbf{p}\in\mathbb{R}^n, \mathbf{q}\in\mathbb{T}^n$, be a n-degree of freedom Hamiltonian system and let us introduce the compact notation $\mathbf{x} = (\mathbf{p},\mathbf{q})\in\mathbb{R}^{2n}$ as well as $\boldsymbol{f} = (-\partial\mathscr{H}/\partial\mathbf{q}, \partial\mathscr{H}/\partial\mathbf{p})\in\mathbb{R}^{2n}$, then the dynamical system is described by the following set of ordinary differential equations

$$\frac{d}{dt}\mathbf{x}(t) = \boldsymbol{f}(\mathbf{x}(t),\boldsymbol{\alpha}), \qquad \mathbf{x}\in\mathbb{R}^{2n}, \tag{2}$$

where $\boldsymbol{\alpha}$ is a vector of parameters entirely defined by the model. Let $\phi(t) = \phi(t;\mathbf{x}_0,t_0)$ be a solution of the flow defined in Eq. (2) with initial conditions $(t_0,\mathbf{x}_0)$, then it has