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Application of evolutionary optimisation to space plasma turbulence modelling

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Abstract

An evolutionary optimisation technique is presented to solve a problem for turbulent space plasma system modelling, using multi-satellite magnetic field measurements of the plasma turbulence.

The application of evolutionary algorithms for system identification allows model structure selection and fitting of parameters for the chosen model using measured inputs and outputs of the system, which can then be used to determine physical characteristics of the system. Genetic algorithms are one such technique that has been implemented. Experimental studies have been performed using multi-point satellite observations providing input and output measurements of the turbulent plasma system. Linear and nonlinear models of the turbulent plasma system are identified and results using genetic algorithms are compared to results obtained from the least squares estimation method.

Keywords: Space plasma; Magnetic field; Multi-point satellite observations; Space physics

1. Introduction

One of the motivations for studying space plasma turbulence in the magnetosheath is the contribution this area of research can make in the understanding of the relationship between the solar wind and Earth's own magnetosphere and so consequently its effect on Earth. Solar activity can disrupt power grids, short wave radio, television and telegraph signals, navigational equipment, defence early warning radar systems, the climate, and can even knockout communication satellites in space in ways that are only now just beginning to be understood.

Attempts to identify processes in space plasma turbulence contained in the magnetosheath have been made using satellite-based time-series measurements of magnetic field waves at and around the region of the bow shock. Analysis of these measurements in the frequency domain has made it possible to construct linear and nonlinear models of the turbulence in this region, which in turn has allowed the investigation of linear and nonlinear processes in the turbulence. Although wave–particle interaction is a linear process, affecting both the wave growth rate and dispersion, through the resulting energy imbalance this process is often responsible for the occurrence of nonlinear processes such as wave–wave interactions that affect coupling between waves within the turbulence.

The aim of this paper is to apply a new technique in the estimation of the linear and nonlinear models of the turbulence, based on genetic algorithms, and assess its performance by comparing the results with those obtained using an existing technique. All the space plasma turbulence measurements used in this paper are taken from those made by the AMPTE UKS and IRM satellites as they crossed the bow shock and passed into the magnetosheath on October 20, 1984.

2. Turbulent space plasma modelling

Amongst many things that need to be known to understand plasma turbulence is a description of the composition

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of the plasma, i.e. which wave modes are observed in the plasma. Knowledge of the processes that occur in the plasma turbulence, i.e. energy transfer, is needed as well. Energy transfer between plasma particles and waves is a linear process that does not involve energy exchange between spectral components. Energy transfer between waves however is a nonlinear process. The Decay Instability, involving three waves, and the Modulational Instability, involving four waves, are two examples of nonlinear processes.

A method to reveal this information exists in the framework of System Identification. A model is constructed from a priori knowledge of the system and then measurements of the plasma system are used to estimate the system parameters. The choice of a suitable model is important so that these estimated parameters can be interpreted in an appropriate physical sense.

2.1. Model structure

A large class of systems can be modelled as Single Input Single Output (SISO). Without loss of generality these systems can be described, in the time domain, by an equation of the form,

$$y(t) = \mathscr{F}(u(t)) \tag{1}$$

where the output of the system $y(t)$ is related to the inputs $u(t)$ to the system through an arbitrary operator $\mathscr{F}$.

Linear systems can be expressed as,

$$y(t) = \int_0^\infty h_1(t)u(t-\tau)\mathrm{d}\tau \tag{2}$$

where $h_1(t)$, the Impulse Response Function, represents fully and completely the dynamics of the linear system. Eq. (2) has a frequency (Fourier) Domain representation,

$$Y(f) = H_1(f)U(f) \tag{3}$$

where $Y(f)$, $U(f)$ are the Fourier transforms of the output and input variables, respectively. $H_1(f)$ is the Fourier transform of the Impulse Response Function, called the Linear Frequency Response Function, and fully represents the dynamics of the linear system in the Frequency domain. Eqs. (2) and (3) are equivalent.

Nonlinear systems can be considered by continuing the expansion of Eq. (2) to include higher order terms,

$$\begin{aligned} y(t) = &\int_0^\infty h_1(\tau)u(t-\tau)\mathrm{d}\tau \\ &+ \int\int_0^\infty h_2(\tau_1,\tau_2)u(t-\tau_1)u(t-\tau_2)\mathrm{d}\tau_1\mathrm{d}\tau_2 \\ &+ \int\int\int_0^\infty h_3(\tau_1,\tau_2,\tau_3)u(t-\tau_1)u(t-\tau_2) \\ &\times u(t-\tau_3)\mathrm{d}\tau_1\mathrm{d}\tau_2\mathrm{d}\tau_3 + \cdots \end{aligned} \tag{4}$$

where $h_i(\tau_1, \ldots, \tau_i)$ are the ith (higher) order generalisations of the Impulse Response Function. This equation has a frequency domain form (Rugh, 1981),

$$\begin{aligned} Y(f) = &H_1(f)U(f) \\ &+ \int_0^\infty H_2(f_1,f_2)U(f_1)U(f_2)\delta(f_1+f_2-f)\mathrm{d}f_1\mathrm{d}f_2 \\ &+ \int\int_0^\infty H_3(f_1,f_2,f_3)U(f_1)U(f_2)U(f_3) \\ &\times \delta(f_1+f_2+f_3-f)\mathrm{d}f_1\mathrm{d}f_2\mathrm{d}f_3 + \cdots \end{aligned} \tag{5}$$

where $H_i(f_1, \ldots, f_i)$ are known as the Generalised Frequency Response Functions (GFRFs) of the system. The delta functions are included to indicate that only frequencies that satisfy $\sum_{j=1}^{i} f_j = f$ contribute to the output, where f is the output frequency and $f_1, \ldots, f_i$ are the input frequencies. If all the Generalised Frequency Response Functions are known then the nonlinear system is fully determined. As in the linear case, Eqs. (4) and (5) are equivalent.

A similar Fourier expansion occurs naturally when considering the Hamiltonian formulation of weak plasma turbulence (Zakharov et al., 1985). In this approach system kernels $\Gamma_i(f_1, \ldots, f_i)$ appear, in a similar way that GFRFs appear in Eq. (5), and analytic expressions for the former can be derived. It is the similarity with the Hamiltonian formulation that makes the frequency domain model presented here a natural choice for modelling plasma turbulence.

2.2. Model parameters

Eq. (5) shows the first, second and third order terms of the system expansion in the frequency domain. There are theoretically an infinite number of terms which practically are impossible to compute so a shortened model is needed. It is not immediately clear which terms in Eq. (5) are significant and in general it is not valid to arbitrarily drop terms. For the case examined here, that of weak plasma turbulence, the assumption is made that nonlinear interactions involving four or more waves are less significant than those involving three or less waves, so the former terms can be dropped and the latter retained.

It should be noted this is not always a safe assumption to make as the relative strength of the nonlinear terms may depend significantly on the local plasma conditions. This is a first assumption with the practical benefit of simplifying the model to one more easily solvable from a computational perspective, at the expense of restricting the model to one allowing interactions involving a maximum of only three waves, perhaps missing some of the physics. Increasing the number of terms in the model is not forbidden by this argument, and is worthy of future study.

The integration in Eq. (5) is over an infinite range of frequencies. Practically however sampling a signal for a finite duration inherently imposes limits on the frequency bandwidth and resolution of the sampled signal. For a signal of length T seconds sampled at a rate f_s Hz (with $n_s = Tf_s$ samples), the limit on the lowest frequency is determined by the length of the signal, $f_{\min} = 1/T$. This is also the Fourier frequency resolution of the signal δf. The highest