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Attitude stabilization of a rigid spacecraft in the geomagnetic field

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Abstract

An analytical method is proposed to study the attitude stability of a triaxial spacecraft moving in a circular Keplerian orbit in the geomagnetic field. The method is developed based on the electrodynamics effect of the influence of the Lorentz force acting on the charged spacecraft's surface. We assume that the rigid spacecraft is equipped with an electrostatic charged protective shield, having an intrinsic magnetic moment. The main elements of this shield are an electrostatic charged cylindrical screen surrounding the protected volume of the spacecraft. The rotational motion of the spacecraft about its centre of mass due to torques from gravitational force, as well Lorentz and magnetic forces is investigated. The equilibrium positions of the spacecraft in the orbital coordinate system are obtained. The necessary and sufficient conditions for the stability of the spacecraft's equilibrium positions are constructed using Lyapunov's direct method. The numerical results have shown that the Lorentz force has a significant influence on the stability of the equilibrium positions, which can affect the attitude stabilization of the spacecraft.

Keywords: Rigid spacecraft; Attitude dynamics; Rotational motions; Lorentz force; Stability; Scientific balloon; Balloon technology

1. Introduction

It is very important to control the interactions of the spacecraft with one or more of various ambient fields, which produce a different disturbance torque. So, the disturbance torque becomes a control torque for the spacecraft. One application of this method is to generate a controlled magnetic moment in a spacecraft, which interacts with the Earth's magnetic field in order to control the spacecraft orientation, or to perform some similar control function.

The geomagnetic field had considerable effect on the spacecraft potential due to magnetic field confinement of the electrons as well as to the electric field resulting from the movement of the spacecraft across magnetic field lines. Anderson et al. (1994) examined the relationship between the plasma environment and spacecraft potential for the Dynamics Explorer 2 (DE 2) spacecraft in an attempt to improve the accuracy of ion drift measurements by the retarding potential analyser (RPA). They derived an algorithm for determining the spacecraft potential (at the location of the RPA on the spacecraft) for any point of the DE 2 orbit.

The important quantity, which determines the magnitude of the effect, is the satellite's electrical charge. The surface of a satellite is charged to a negative potential (Al'pert et al., 1964) and in the first approximation behaves like a spherical condenser with respect to the ionosphere vicinity.

Several methods were developed to study the attitude control of the magnetic fields (Jan and Tsai, 2005; Silani and Lovera, 2005).

Chen and Liu (2002) and Chen et al. (2002) investigated the chaotic motion of a magnetic rigid spacecraft and its control with internal damping in a circular orbit near the equatorial plane of the Earth.

A series of papers were already made to assess the effects of Lorentz force on the orbital motion (Sehnal, 1969; Ciufolini, 1987; VokRouhlicky, 1989; Antal and Mihály, 1997;

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Peck Mason, 2005; Abdel-Aziz, in press). On the other hand there is some publications studied the Lorentz force influence on the dynamic of the rotational motion of the satellite (Beletskii and Khentov, 1985; Tikhonov, 1990). Lanoixo et al. (2005) have been shown that the Lorentz forces can be effectively used to decay the orbit of a satellite. For the particular control system described in their paper, Lorentz forces are more effective for orbital decay than chemical rocket propulsion when the satellite mass is larger than approximately 100 kg.

Yehia (2001) have been recently shown that the motion involving electrically charged and magnetized rigid bodies under certain combinations of gravitational, electric and Lorentz electromagnetic forces are integrable problem.

In the present work a rigid spacecraft, which equipped with electrostatic charged protective shield is considered, the main element of this shield is an electrostatic charged cylindrical screen surrounding the protected volume of the spacecraft. The rotational motion of the spacecraft about its centre of mass due to torque from gravitational force, as well as Lorentz and magnetic forces is investigated.

Possible equilibrium positions of the spacecraft in the orbital coordinate system are obtained. The necessary and sufficient conditions for the stability of the spacecraft's equilibrium positions are constructed using Lyapunov's direct method.

2. The equations of motion

A rigid spacecraft is considered whose centre of mass moves in the Newtonian central gravitational field of the earth in a circular orbit of radius R. We suppose that the spacecraft is equipped with an electrostatic charged protective shield, having an intrinsic magnetic moment. The main element of this shield is an electrostatic charged cylindrical screen surrounding the protected volume of the spacecraft. The rotational motion of the spacecraft about its centre of mass is analysed, considering the influence of gravity gradient torque $\boldsymbol{T}_{\mathrm{G}}$ and the torques $\boldsymbol{T}_{\mathrm{L}}$ and $\boldsymbol{T}_{\mathrm{M}}$ due to Lorentz and magnetic forces, respectively. The torque $\boldsymbol{T}_{\mathrm{L}}$ results from the interaction of the geomagnetic field with the charged screen of the electrostatic shield.

Let $OX'Y'Z'$ and $OX_0Y_0Z_0$ be two coordinate systems with a common origin O at the spacecraft's centre of mass. $OX'Y'Z'$ is the orbital coordinate system with OX' tangent to the orbit in the direction of motion, OZ' lies along the normal to the orbital plane, and OY' lies along the radius vector R of the point O relative to the centre of the Earth. $OX_0Y_0\,Z_0$ is the system of principal central axes of inertia of the spacecraft. Let θ, ψ and ϕ be the Eulerian angles such that the angle of precession ψ is taken in plane orthogonal to Z_0, θ is the notation angle between Z', Z_0 and ϕ is the angle of self-rotation around the Z'-axis. Let A, B, C are the principal moments of inertia of the spacecraft.

Let a uniform magnetic field $\boldsymbol{H} = H(\beta_1, \beta_2, \beta_3)$ be directed along the normal of the orbit, where H is the magnitude of the intensity of the magnetic field, with the total magnetic moment $\boldsymbol{M} = M(0, 0, 1)$ directed along the Z'-axis, where M is the magnitude of the magnetic moment.

The components of the Lorentz force, will be the components of a vector (Sehnal, 1969; Peck Mason, 2005)

$$\boldsymbol{F}_{\mathrm{L}} = Q\boldsymbol{V} \times \boldsymbol{H}, \tag{1}$$

where $\boldsymbol{V}$ is the velocity vector of the satellite in the orbit, Q is the satellite's electrical charge.

The spacecraft is supposed to be equipped with a charged surface. According to Beletskii and Khentov (1985), the matrix S of the electrostatic charged cylindrical screen takes the form:

$$S = \begin{pmatrix} Db^2 & 0 & 0 \\ 0 & Db^2 & 0 \\ 0 & 0 & Da^2 \end{pmatrix}, \tag{2}$$

where $D = \frac{QH}{4R^3}$, $2a$ and $2b$ are the length and diameter of the cylindrical shell.

In the orbital system, the torque $\boldsymbol{T}_{\mathrm{L}}$ due to the Lorentz force, which can affect the spacecraft attitude, takes the following expression (Beletskii and Khentov, 1985):

$$\boldsymbol{T}_{\mathrm{L}} = \boldsymbol{\omega} \times \boldsymbol{\beta S} + \boldsymbol{\omega}_{\mathrm{g}} \times \boldsymbol{\beta S}, \tag{3}$$

where, $\boldsymbol{\omega}$ is the angular velocity vector of the spacecraft, $\boldsymbol{\omega}_{\mathrm{g}} = \omega_{\mathrm{g}}\boldsymbol{\beta}$ is the vector of angular velocity of the diurnal rotation of the geomagnetic field together with the Earth.

The equations of motion of a rigid spacecraft are usually written in the Euler–Poisson variables $\boldsymbol{\omega}$, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ and have the following form (Wertz, 1978):

$$\dot{\boldsymbol{\omega}} I = -\boldsymbol{\omega} \times \boldsymbol{\omega} I + \boldsymbol{T}_{\mathrm{G}} + \boldsymbol{T}_{\mathrm{M}} + \boldsymbol{T}_{\mathrm{L}}, \tag{4}$$

$$\dot{\boldsymbol{\alpha}} + \boldsymbol{\alpha} \times \boldsymbol{\omega} = -\Omega\boldsymbol{\gamma}, \quad \dot{\boldsymbol{\beta}} + \boldsymbol{\beta} \times \boldsymbol{\omega} = 0, \quad \dot{\boldsymbol{\gamma}} + \boldsymbol{\gamma} \times \boldsymbol{\omega} = \Omega\boldsymbol{\alpha}, \tag{5}$$

where

$$\boldsymbol{T}_{\mathrm{G}} = 3\Omega^2 \boldsymbol{\gamma} \times \boldsymbol{\gamma} I, \quad \boldsymbol{T}_{\mathrm{M}} = \boldsymbol{M} \times \boldsymbol{H}, \tag{6}$$

$I = \mathrm{diag}\,(A, B, C)$ is the inertia matrix of the spacecraft, Ω is the orbital angular velocity. $\boldsymbol{\alpha}$, $\boldsymbol{\gamma}$, $\boldsymbol{\beta}$, are the unit vectors along the axes of the orbital coordinate system. These vectors are the different directions of the tangent to plane of the orbit, its radius and the normal of the orbit, respectively (Yehia, 2001),

$$\begin{aligned}
\boldsymbol{\alpha} &= (\alpha_1, \alpha_2, \alpha_3) \\
&= (\cos\psi\cos\phi - \sin\psi\sin\phi\cos\theta,\ -\cos\psi\sin\phi \\
&\qquad - \cos\theta\sin\psi\cos\phi,\ \sin\theta\sin\psi), \\
\boldsymbol{\beta} &= (\beta_1, \beta_2, \beta_3) \\
&= (\sin\psi\cos\varphi + \cos\theta\cos\psi\sin\phi,\ -\sin\psi\sin\phi \\
&\qquad + \cos\theta\cos\psi\cos\phi,\ -\sin\theta\cos\psi), \\
\boldsymbol{\gamma} &= (\gamma_1, \gamma_2, \gamma_3) \\
&= (\sin\theta\sin\phi,\ \sin\theta\cos\phi,\ \cos\theta).
\end{aligned} \tag{7}$$

The angular velocity of the spacecraft can be written as

$$\boldsymbol{\omega} = (p, q, r) = \dot{\psi}\boldsymbol{\gamma} + \dot{\theta}\boldsymbol{n} + \dot{\phi}\boldsymbol{\delta}, \tag{8}$$