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Numerical study of the time required for the gravitational capture in the bi-circular four-body problem

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Abstract

A gravitational capture occurs when a spacecraft (or any particle with negligible mass) changes from a hyperbolic orbit with a small positive energy around a celestial body into an elliptic orbit with a small negative energy without the use of any propulsive system. The force responsible for this modification in the orbit of the spacecraft is the gravitational force of the third and the fourth bodies involved in the dynamics. In this way, those forces are used as a zero cost control, equivalent to a continuous thrust applied in the spacecraft. One of the most important applications of this property is the construction of trajectories to the Moon to minimize fuel consumption. The concept of gravitational capture is used, together with the basic ideas of the gravity-assisted maneuver and the bi-elliptic transfer orbit, to generate a trajectory that requires fuel consumption smaller than the one required by the Hohmann transfer. The objective of the present paper is to study the time required for the ballistic gravitational capture in a dynamical model that has the presence of four bodies. In particular, the Earth–Moon–Sun–Spacecraft system is considered.

Keywords: Astrodynamics; Orbital maneuvers; Gravitational capture; Bi-circular four-body problem; Time of capture

1. Introduction

The application of the gravitational capture phenomenon in spacecraft trajectories is a very important technique in astrodynamics. Among the first studies are the ones performed by Belbruno (1987, 1990, 1992a,b, 2002), Krish et al. (1992), Miller and Belbruno (1991), Belbruno and Miller (1990a,b, 1993). They all studied missions in the Earth–Moon system that use this technique to save fuel during the insertion of the spacecraft in its final orbit around the Moon. Another group of researches that made fundamental contributions in this field, also with the main objective of constructing real trajectories in the Earth–Moon system, is the Japanese group that includes the publications Yamakawa et al. (1992, 1993). In particular, Yamakawa wrote his Ph.D. dissertation (Yamakawa, 1992) on this topic, with several important contributions in this field. Krish (1991) and Krish et al. (1992) also considered those transfers to the Moon. A real application of those ideas was made during an extended phase of a Japanese spacecraft (Belbruno and Miller, 1990a,b). A study of this problem, from the perspective of invariant manifolds and using a four-body model, was developed by Belbruno (1994). After that, some studies that consider the time required for this transfer appeared in the literature. Examples of this approach, using the dynamical model of the restricted three-body problem, can be found in the papers by Vieira Neto and Prado (1995, 1998). An extension of the dynamical model to consider the effects of the eccentricity of the primaries is also available in the literature, like in Vieira Neto and Prado (1996). Routes to the Moon were also analyzed by Bello-Mora et al. (2000) and Circi and Teofilatto (2001). Recently, a new excellent book was published dedicated to the gravitational capture phenomenon

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by Belbruno (2004). Analytical approximations can be found in Prado (2002), as well as in Prado (2006), that considered the problem of gravitational capture under the restricted four-body problem in terms of the used in Leiva and Briozzo (2005), to search for fast periodic transfer orbits in the Sun–Earth–Moon system. The orbits found in this paper perform periodic Earth–Moon transfers with a period of approximately 29.5 days. The same goal of finding low cost trajectories to the Moon is used by De Melo and Winter (2006), where a region of direct stable orbits around the Moon is investigated, and whose stability is related to the H2 family of periodic orbits and to the quasi-periodic orbits that oscillate around them. The use of invariant manifolds to perform ballistic gravitational capture is another important technique, like appears in Koon et al. (2001), where trajectories to the Moon are analyzed in a system where the primary bodies are the Sun, the Earth and the Moon.

The bi-circular problem (Simo et al., 1995; Castella and Jorba, 2000) is a particular case of the problem of four bodies, where one of the masses, let us say m_4, is supposed to be infinitely smaller than the other three masses. With that hypothesis, m_4 moves under the gravitational forces of m_1, m_2 and m_3, but it doesnot disturb the motion of the three bodies with significant mass. In the bi-circular problem, the motion of m_1, m_2 and m_3 around the center of mass is considered as formed by circular orbits and the motion of m_4 has to be a certain function of the initial conditions. We can consider the bi-circular problem as a disturbance of the restricted problem of three bodies, with the presence of one more body in circular orbit. This problem can be used as a model for the motion of a space vehicle in the Sun–Earth–Moon system.

In the first part of the present paper we supplied the equations of motion of the model and we defined gravitational capture. The second part is used for the calculation of some numerical results for the bi-circular problem, such as direct orbits, retrograde orbits, capture orbits, etc.

2. Mathematical models

The problem of four bodies with the two hypotheses shown below is called "bi-circular problem" and it is shown in Fig. 1.

First hypothesis. It is considered the existence of two bodies with significant mass moving in circular orbits around the mutual center of mass. Those two bodies are called primaries.

Second hypothesis. The third body, with significant mass, is in a circular orbit around the center of mass of the system formed by the two first primaries and its orbit is coplanar with the orbits of those primaries.

The goal is to study the motion of a fourth body, with negligible mass, under the gravitational attractions of the three bodies with significant mass. See Fig. 1 for details.

The planar equations of motion of the space vehicle in the sidereal and synodical systems are shown below. We use the canonical system of units, by dividing all the distances by the distance between the two primaries and dividing all the masses by the total mass of the two primaries. It is also defined that the angular velocity of the primaries is unitary. The masses and distances of the Earth, Moon and Sun are: Mass of the Earth, $M_E = 5.98 \times 10^{24}$ kg; Mass of the Moon, $M_M = 7.35 \times 10^{22}$ kg; Mass of the Sun, $M_S = 1.99 \times 10^{30}$ kg. Earth–Moon distance $d_1 = 3.844 \times 10^5$ km; Earth–Sun distance $d_2 = 1.496 \times 10^8$ km.

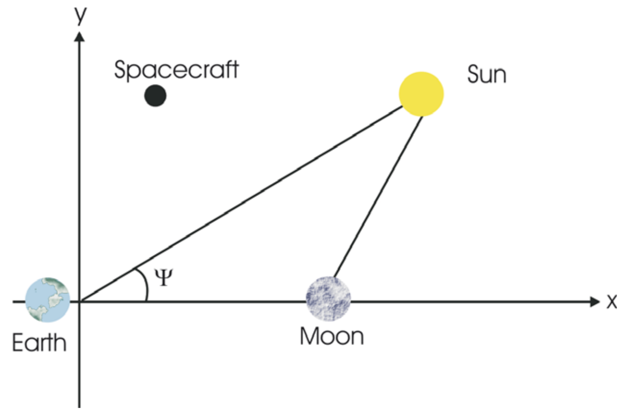

Fig. 1. Restricted four-body model (Cartesian Coordinate).

Then, the masses of the Earth, Moon and Sun in the canonical system are:

$$\text{Mass of the Earth} = \mu_E = \frac{M_E}{M_M + M_E} = 0.9878715$$

$$\text{Mass of the Moon} = \mu_M = \frac{M_M}{M_E + M_M} = 0.0121506683$$

$$\text{Mass of the Sun} = \mu_S = \frac{M_S}{M_E + M_M} = 328900.48.$$

The circumferences described by the Moon and the Earth has radius μ_E and μ_M, respectively. (x, y), (x_E, y_E), (x_M, y_M) and (x_S, y_S) are the sidereal coordinates of the space vehicle, the Earth, the Moon and the Sun, respectively. Below are the equations of motion of the Earth, Moon and Sun: $x_E = -\mu_M \cos(t)$, $y_E = -\mu_M \sin(t)$, $x_M = \mu_E \cos(t)$, $y_M = \mu_E sin(t)$, $x_S = R_S \cos(\psi)$, $y_S = R_S \sin(\psi)$ and $\psi = \psi_0 + \omega_S t$,

where $R_s = 389.1723985$ is the distance between the Sun and the center of the system, $\omega_s = 0.07480133$ is the angular velocity of the Sun, ψ is the angle that the Sun makes with the horizontal axis, ψ_0 is the initial value of ψ and t is the time.

The distance of the space vehicle to the Earth is $r_1 = \sqrt{(x - x_E)^2 + (y - y_E)^2}$; to the Moon is $r_2 = \sqrt{(x - x_M)^2 + (y - y_M)^2}$; to the Sun is $r_3 = \sqrt{(x - x_S)^2 + (y - y_S)^2}$.

Therefore, we have the equations of motion of the space vehicle in the inertial system: