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Nusselt number for convection driven by tidal and radiogenic heating

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Abstract

The 'traditional' formulas giving the Nusselt number *Nu* as a function of Rayleigh number *Ra* cannot be used for low and moderate values of *Ra*. Moreover, the recent progress in 3D numerical modeling makes possible to determine the Nusselt number *Nu* as a function of Rayleigh number *Ra* for convection driven by radiogenic and tidal heating. We found that for low and moderate *Ra*: $Nu(Ra) = \varepsilon(Ra+\xi)^{\lambda}$ where λ depends on rheology and boundary conditions, ε depends only on the mode of heating, and $\xi = \varepsilon^{-1/\lambda} \cdot Nu(Ra)$ makes possible to develop a parameterized theory of convection in medium size icy satellites. We also indicate some differences between convection driven by tidal and radiogenic heating and convection driven exclusively by radiogenic heating.

Keywords: Satellites of Saturn; Tides; Thermal histories; Tectonics; Volcanism

1. Introduction

The role of solid state convection driven mainly by radiogenic heat is commonly accepted for the Earth (e.g. Czechowski, 1993). Many global features observed on some medium-sized icy satellites (MIS) of Saturn and Uranus could be interpreted also as results of solid state convection (Schubert et al., 1986; McKinnon, 1998; Pater and Lissauer, 2001, p. 203). The main difference is the source of heat; for some of MIS (e.g. Enceladus), the heating from tidal deformation is (or was) probably more important than the radiogenic or primordial heating. It is confirmed by correlation of tectonic activity and tidal parameter found by Czechowski and Leliwa-Kopystyński (2005).

Thermal and dynamical properties of convection driven by tidal and radiogenic heating are significantly different than the properties of convection driven exclusively by radiogenic heating for low and moderate Rayleigh number *Ra*. Moreover, this range of *Ra* is not covered by 'traditional' formulas that are obtained for high *Ra* (e.g. Rossby, 1969; Christensen, 1984; Solomatov, 1995). Let us consider the following formula developed for a fluid layer heated from below with no-slip wall boundary (i.e. rigid boundary conditions):

$$Nu(Ra) = 0.131Ra^{0.3} \tag{1}$$

(Turcotte and Schubert, 1982, Ch. 6). A simple calculation indicates that for $Ra < 875$ this formula gives unphysical value ($Nu < 1$). For a little higher *Ra* it gives $Nu > 1$ but still *Nu* is underestimated. The above formula could be used for high *Ra* only (let us say for $Ra > 10^5$). This is not a drawback if the formula is used for large satellites like Io or Europa (where $Ra > 10^6$) but is a drawback if it is used for medium-sized satellites where *Ra* is low or moderate. Note that Mitri and Showman (2005) in the paper concerning Europa consider the Nusselt number for *Ra* in the range 10^6–10^{10} (the lower limit for Europa's shell is close to the higher limit for MIS!). The higher *Ra* in Europa (comparing to MIS) is a result of a few factors: temperature (in most of MIS the temperature is lower, i.e. viscosity is higher), and the gravity. There are also some differences in definition of *Ra*. The low values of *Ra* in MIS means that 'traditional' expressions of the Nusselt number as a function of the Rayleigh number *Ra* cannot be used for MIS.

The main aim of the present paper is to develop function *Nu*(*Ra*) for the solid state convection driven by tidal and

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radiogenic heating for low and moderate Ra (let us say for $0 < Ra < 10^6$). This function could be used in the parameterized theory of convection in MIS (e.g. Czechowski, 2006a,b).

The recent data from Cassini mission indicate that most of the MIS of Saturn could be treated approximately as homogeneous solid bodies (Thomas et al., 2007). Therefore we consider here the solid state convection in a homogeneous body heated by radioactive elements and tidal deformations. Of course, some crio-magma chambers could exist but we assume that these chambers are not important for global properties of convection. Some authors indicate a possibility of existence of a subsurface ocean of liquid water. A recent paper by Roberts and Nimmo (2008) indicate that such pure water ocean in Enceladus would freeze in 30 Ma and "even a water-ammonia eutectic composition will only prolong the freezing, not prevent it" (compare also with Schubert et al., 2007). In larger satellites the ocean could be stable but Hussmann et al. (2006) have found that subsurface ocean is possible only in one Saturnian MIS (Rhea) and two Uranian MIS (Titania and Oberon). It means that for the rest of MIS our assumptions are valid.

2. Dimensionless numbers *Ra* and *Nu*

Full description of convection is given by velocity field and temperature distribution. However, for many applications the full description is not necessary and a given property of thermal convection could be described by a few dimensionless numbers. Rayleigh number Ra is used to characterize intensity of convection. For convection in a sphere, Ra is defined as (e.g. Czechowski and Leliwa-Kopystyński, 2005):

$$Ra = \rho_0^2 g_0 \alpha Q_{av} d^5 / (k \eta_0 \kappa), \quad (2)$$

where ρ_0 is the reference density [kg m^{-3}], g_0 is the surface gravity [m s^{-2}], α is the coefficient of thermal expansion [K^{-1}], d is the radius of the body [m], k is the coefficient of thermal conductivity [W m^{-1} K^{-1}], η_0 is the reference viscosity [Pa s], κ is the coefficient of thermal diffusion [m^2 s^{-1}] (note that: $k = \kappa c \rho_0$ where c is the specific heat [J kg^{-1}]). Rate of heating is given by Q_{av} [W kg^{-1}]. Note that the viscosity η_0 and the heat production Q_{av} in Eq. (2) should be treated as effective values because they depend on temperature and position.

The range of possible values of Ra cannot be calculated from (2) because we do not know the viscosity η_0 and efficiency of tidal heating Q_{av}. Viscosity depends on many unknown factors like: temperature, grain size, admixtures of ammonia and possibly of other volatiles (e.g. Prentice, 2005 and the discussion below). We can infer a range of possible Ra from the observational data. The lack of plate tectonics indicates that convection is (and was) rather weak (i.e. low Ra). On the other hand some global tectonic features (like Ithaca Chasma on Tethys) indicate episodes when convection was strong enough to break the lithosphere (i.e. moderate Ra). The global patterns of tectonic features also indicate low or moderate values of Ra corresponding to 1-cell or 2-cell convection pattern (Czechowski and Leliwa-Kopystyński, 2005). Note that even the most active MIS (Enceladus) has obvious dichotomy (volcanic activity concentrates around the South Pole) indicating a simple pattern of convection below the crust.

In a case of highly symmetric physical situation, e.g. in a planet with a spherically symmetric distribution of heat production, convection starts only if Ra exceeds some critical value $Ra_{cr} > 0$. For asymmetric heating convection exists even for very low Ra (but $Ra > 0$) and this fact is expressed by stating that $Ra_{cr} = 0$ (e.g. Czechowski and Leliwa-Kopystyński, 2005).

Nusselt number Nu characterizes the efficiency of convective heat transport. If heat production is given, Nu is defined as:

$$Nu = (\Delta T_{cond} / \Delta T_{conv}) \quad (3)$$

where ΔT_{cond} and ΔT_{conv} are temperature differences for the same parameters. ΔT_{cond} denotes the average temperature difference without convection (i.e. while convection is artificially prohibited and conduction is the only process of heat transfer) and ΔT_{conv} denotes the true average temperature difference (with a possible convection). Of course for $Ra = 0$ there is no convection so $\Delta T_{cond} = \Delta T_{conv}$ and $Nu = 1$.

In the present paper, the average temperature difference is defined as: $\Delta T = T_{av} - T_s$ where T_{av} is the globally averaged temperature and T_s is the surface temperature. Without convection, Nu equals to 1. With convection Nu increases with increasing Ra. High values of Nu mean more efficient cooling in respect to the cooling without convection. According to Schubert et al. (2001, p. 379): "[...] Nusselt number [...] is a particularly important diagnostic, and its dependence on Ra [...] has been a subject of numerous investigations". Note also that Nu is used for steady or quasi-steady convection.

The exact value of Nu is determined by the physical fields that describe the convection itself. Fortunately, for most applications we need only some approximation of Nu. This approximation is usually expressed as a simple function of Ra (e.g. Eq. (1)). Additional parameters could be used to characterize the dependence of viscosity on temperature like in the following equation (Morris and Canright, 1984; Fowler, 1985; Mitri and Showman, 2005):

$$Nu(Ra) = \gamma Ra^a \chi^y \quad (4)$$

where γ, y, a are determined by theoretical calculation, laboratory or numerical experiments (see discussion in Section 1, Turcotte and Schubert, 1982, p. 285; Schubert et al., 2001, p. 381) and $\chi = -\partial \ln \eta / \partial T$. Unfortunately, in some situations, Eq. (4) still could give unphysical values for low Ra. Therefore the above approximation (as well as other similar approximations, e.g. Solomatov, 1995) is of lesser value for convection in MIS (i.e. for global convection driven by tidal heating for low and moderate Ra) be-