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Non-linear diffusion of cosmic rays

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Abstract

The propagation of cosmic rays in the interstellar medium after their release from the sources – supernova remnants – can be attended by the development of streaming instability. The instability creates MHD turbulence that changes the conditions of particle transport and leads to a non-linear diffusion of cosmic rays. We present a self-similar solution of the equation of non-linear diffusion for particles ejected from a SNR and discuss how obtained results may change the physical picture of cosmic ray propagation in the Galaxy.

Keywords: Cosmic rays; Supernova remnants; Interstellar turbulence; Streaming instability

1. Introduction

The cosmic rays are relativistic charged particles and they not always can be considered as test particles moving in given regular and random interstellar magnetic fields (Ginzburg, 1966). In particular, the cosmic ray streaming instability may amplify the MHD waves in background plasma and affect the level of interstellar turbulence that controls particle scattering on pitch-angle and determines the cosmic ray transport. The self-consistent approach is needed when this collective effect is essential (Skilling, 1971). This is the situation in the studies of diffusive shock acceleration in SNRs. The acceleration is accompanied by strong streaming instability of cosmic rays at the shock precursor region (Bell, 1978, 2005; McKenzie and Völk, 1982; Lucek and Bell, 2000). The general leakage of cosmic rays from the Galaxy can also occur with a development of streaming instability (Wentzel, 1969; Kulsrud and Pearce, 1969; Kulsrud and Cesarsky, 1971; Holmes, 1975; Ptuskin et al., 1997; Farmer and Goldreich, 2004). In the present work, we consider the intermediate stage of cosmic ray propagation when the cloud of energetic particles accelerated after a SN burst has left the source but not yet completely mixed with the background cosmic rays produced by other SNR. The point that is essential for our consideration is a relatively large gradient of cosmic ray density within a few hundred parsecs of the source that leads to the development of streaming instability.

2. Non-linear diffusion

The propagation of cosmic rays with energies 10^9–10^{15} eV is usually described in the diffusion approximation; see Berezinskii et al. (1990). The simplified equation for cosmic ray diffusion coefficient along the average magnetic field at the resonant scattering of particles by random magnetic field is $D = \frac{4vr_g}{3\pi U(k_r)}$, where $v \approx c$, r_g, and $k_r = 1/r_g$ are the particle velocity, the particle Larmor radius, and the resonant wave number respectively, see Bell (1978). The function $U(k)$ characterizes the spectral distribution of random magnetic field: $(\delta B)^2 = B^2 \int \frac{dk}{k} U(k)$, where the average magnetic field $B = 5 \times 10^{-6}$ G and $B \gg \delta B$.

In the problem considered in the present work, the diffusion flux of cosmic rays amplifies the amplitude of random resonant MHD waves with the growth rate $\Gamma_{cr}(k_r) = \frac{16\pi^2 V_a v p^4}{3B^2 U(k_r)} \nabla f$ (e.g. Lagage and Cesarsky, 1983), where it is assumed that the instability threshold is significantly exceeded, V_a is the Alfven velocity in the interstellar medium, and $f(t, \mathbf{r}, p)$ is the particle distribution function on

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momentum p (the total density of cosmic ray particles is $4\pi\int dpp^2 f$).

The wave amplification is balanced by the wave dissipation with some decrement $\Gamma_{\rm dis}$. The selection of the appropriate mechanism of dissipation that determines $\Gamma_{\rm dis}$ is ambiguous because of the diversity and complexity of non-linear processes of wave interactions in the magnetized space plasma. To be specific, we assume the Kolmogorov type of non-linear dissipation for the MHD turbulence and use a simple equation for the efficiency of non-linear dissipation $\Gamma_{\rm K}(k) = (2C_{\rm K})^{-3/2} kV_{\rm a}\sqrt{U(k)}$, where $C_{\rm K} \approx 3.6$, see Ptuskin and Zirakashvili (2003).

The steady state condition $\Gamma_{\rm cr} = \Gamma_{\rm dis}$ allows to obtain the self-consistent expression for diffusion coefficient and to write down the corresponding non-linear equation of diffusion in the following form:

$$\frac{\partial f}{\partial t} - \nabla D \nabla f = 0, \quad D = \frac{\kappa}{|\nabla f|^{2/3}}, \tag{1}$$

where $\kappa = \frac{(vr_g)^{1/3} B^{4/3}}{2^{5/3} 3^{1/3} \pi^{7/3} C_{\rm K} p^{8/3}}$. Here we ignore the contribution of background interstellar turbulence produced by external sources and neglect the possible convective transport of cosmic rays.

Let us analyze a one-dimensional propagation of energetic particles ejected in a tube of magnetic field lines of cross-section S from a SNR at the moment $t = 0$. The magnetized particles move along magnetic field lines symmetrically in both directions from the source located at $x = 0$ (the coordinate x is directed along the regular magnetic field). We consider the static magnetic field which is constant in absolute magnitude and may have a considerable random component. In this case the total cross-section S of a magnetic flux tube remains constant ($S = \text{const}$) along its length x as a consequence of the equation $\nabla\mathbf{B} = 0$ but the tube geometry evolves. The tube experiences continuous shape evolution stretching in one direction and contracting in other, see Rechester and Rosenbluth (1978). The wandering of initially close magnetic field lines leads to their divergence and eventually destroys the tube of correlated field lines over distance of the order of $X_{\rm c} \sim L/A^2 \sim 200$ pc, where $L \sim 100$ pc is the main scale of the interstellar random field and $A^2 \sim 0.5$ is the squared ratio of the random to the total magnetic field strength (here the extended spectrum of random field at scales smaller then L was assumed; the value of $X_{\rm c}$ increases if the random field is concentrated solely at the main scale L). Thus the concept of magnetic flux tube can be used up to scales of a few hundred parsecs.

The conserved number of particles in a flux tube at a given particle momentum is equal to ηS, where $\eta = 2\int_0^\infty dxf$. We search for a self-similar solution of non-linear Eq. (1) in the form $f = \frac{\eta}{\kappa^{3/2} t^{3/2}} \Phi(z)$, where the variable $z = \frac{\eta x}{\kappa^{3/2} t^{3/2}}$ is introduced, and the function $\Phi(z)$ can be determined by solving Eq. (1). As a result, we obtain the following solution for the cosmic ray distribution function in the magnetic flux tube:

$$f(p,x,t) = \frac{1}{\sqrt{\frac{\Gamma^8(1/4)}{3^3\Gamma^4(1/2)} \cdot \frac{\kappa^3 t^3}{\eta^4} + \frac{3^3}{2^4} \cdot \frac{x^4}{\kappa^3 t^3}}}, \tag{2}$$

where $\Gamma(s) = \int_0^\infty dt e^{-t} t^{s-1}$ is the Euler's gamma function.

The solution (2) shows that an observer at the position $x = x_*$ registers the maximum of cosmic ray intensity at the moment $t = t_*$ determined by the equation $x_* = \frac{2\Gamma^2(1/4)}{3^{3/2}\Gamma(1/2)} \cdot \frac{\kappa^{3/2}}{\eta} t_*^{3/2}$. This law of particle propagation differs from the case of an ordinary diffusion with constant diffusion coefficient when $x_* \sim t_*^2$.

Using Eq. (2), one can find the diffusion coefficient:

$$D = \kappa \left|\frac{\partial f}{\partial x}\right|^{-2/3} = \frac{2^2\Gamma^8(1/4) \cdot \kappa^6 t^5}{3^5\Gamma^4(1/2) \cdot \eta^4 x^2} + \frac{3x^2}{4t}. \tag{3}$$

The expression on the right-hand side of Eq. (3) reaches its' minimum value $D_{**} = \frac{3\Gamma^{4/3}(1/4) \cdot \kappa x_{**}^{4/3}}{2^{1/3} 5^{5/6} \Gamma^{2/3}(1/2) \cdot \eta^{2/3}}$ at $x_{**} = \frac{2 \cdot 5^{1/4} \Gamma^2(1/4) \cdot \kappa^{3/2}}{3^{3/2}\Gamma(1/2) \cdot \eta} t_{**}^{3/2}$; the last relation between x_{**} and t_{**} differs little from the relation between x_* and t_*.

The solutions (2) and (3) obtained above for f and D were derived for the burst of a single source (SNR) in an infinite magnetic flux tube. With the multiple random SNR bursts, there exists some region limited by the characteristic size Δx and the characteristic time Δt around every source where this source dominates i.e. where it determines the value of cosmic ray gradient $\frac{\partial f}{\partial x}$. Let us assume that the SN bursts occurs in the galactic disc of the total thickness $2H (H = 150\text{pc}, H \gg \sqrt{S})$ with the frequency $\nu_{\rm sn}$ per unit disk area (we suppose that $\nu_{\rm sn} = 50$ (kpc^2 Myr)$^{-1}$ in the Solar System vicinity). The rough estimate gives the relation $\nu_{\rm sn} H^{-1} S \Delta x \Delta t = 1$ (see Lagutin and Nikulin, 1995 for mathematical analysis of the problem of determination of Δx and Δt in the ensemble of random SN bursts).

The effective diffusion coefficient $D_{\rm ef}$ created in the galactic disk by the streaming instability of cosmic rays released from many randomly distributed SNRs can be estimated as $D_{\rm ef} \approx D_{**}(x_{**} = \Delta x, t_{**} = \Delta t)$ that results in the evaluation

$$D_{\rm ef} \approx \left(\frac{6\Gamma^{12}(1/4)}{5^{7/2}\Gamma^6(1/2)} \frac{H^4\kappa^9}{\nu_{\rm sn}^4 S^4 \eta^6}\right)^{1/5}. \tag{4}$$

The one-dimensional diffusion at the scales $\leqslant \Delta x$ was assumed in the derivation of Eq. (4) that requires the condition $\Delta x \leqslant X_{\rm c}$ to be fulfilled. If the opposite condition $\Delta x > X_{\rm c}$ is valid, the one-dimensional diffusion switches to the three-dimensional one at distance $X_{\rm c}$ that leads to the rapid dilution of cosmic rays expelled from a given SNR in the background cosmic rays produced by other numerous SNR. The generated effective diffusion coefficient in the galactic disk is estimated in this case as $D_{\rm ef,c} \approx D(x = X_{\rm c}, t = \Delta t_{\rm c})$ where $\Delta t_{\rm c}$ is determined by the equation $\nu_{\rm sn} H^{-1} S X_{\rm c} \Delta t_{\rm c} = 1$. It gives then

$$D_{\rm ef,c} \approx \frac{4\Gamma^8(1/4)}{3^5\Gamma^4(1/2)} \frac{H^5\kappa^6}{\nu_{\rm sn}^5 S^5 X_{\rm c}^7 \eta^4}. \tag{5}$$