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Influence of ionospheric conductivity on the ballooning modes in the inner magnetosphere of the Earth

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Abstract

The generation of ballooning perturbations in the inner magnetosphere of the Earth was investigated in the dipole model of the geomagnetic field taking into account the ionospheric boundary conditions. The ionosphere is considered as a thin layer with finite conductivity. A special attention was paid to the study of the influence of ionospheric conductivity on the stability of ballooning perturbations. It was demonstrated that in the case of insulating ionosphere, flute perturbations are generated and they determine the stability of magnetospheric magnetohydrodynamic modes. An analytical stability criterion was obtained for these perturbations. Conforming to the results of the paper (Hameiri, E., Ballooning modes on open magnetic field lines. Phys. Plasmas 6(3), 674–685, 1999), it also determines the stability boundary of magnetospheric magnetohydrodynamic perturbations in the resistive boundary case. In the case of resistive ionosphere, stable toroidal Alfvén waves are slowly decaying. The spectrum of coupled poloidal Alfvén and slow magnetosonic eigenmodes was also investigated for perfectly conductive ionosphere.

Keywords: Space Plasma; Plasma Stability; Inner Magnetosphere; Ionospheric Conductivity; Dipole Configuration; Ballooning Modes; Flute Modes; Alfvén Modes; Magnetosonic Modes

1. Introduction

In this work, we analyze the problems of determination of spectrum of eigenoscillations and the stability boundary of magnetohydrodynamic (MHD) perturbations in the inner magnetosphere of the Earth taking account of finite ionospheric conductivity. These perturbations are generated by the gradient of pressure and are usually called ballooning modes. Many authors investigated these modes before (see e.g. (Liu, 1997) and references therein). Most of earlier works (Ivanov and Pokhotelov, 1987), (Hameiri et al., 1991), (Klimushkin, 1997) pointed out a principal dependence of oscillation spectrum and stability boundary of ballooning perturbations on ionospheric conductivity. A significant progress in understanding of the influence of ionospheric conductivity on the ballooning stability was achieved in the work (Hameiri, 1999). The most important result of this paper is a statement that ‘‘a ballooning instability occurs for resistive bounding ends if, and only if, it occurs when the ends are insulators’’. This result suggested us the possibility of deriving an analytical stability criterion for the magnetospheric ballooning modes with realistic boundary conditions. This task is completed in this work (see Section 6). Moreover, we investigated toroidal Alfvén modes with resistive boundary (Section 5) and coupled poloidal Alfvén and slow magnetosonic modes with perfectly conducting boundary (Section 7).

Just like our previous work (Cheremnykh et al., 2004), we describe perturbations in the framework of a well-known model of an axially symmetric dipolar magnetic field and, following Cheng (1992), assume the

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magnetospheric plasma equilibrium to be provided by a toroidal current. We describe ballooning perturbations with the equations of small oscillations, derived by us earlier in the paper (Cheremnykh et al., 2004) in eikonal approximation (Dewar and Glasser, 1983). To obtain boundary conditions, taking into account finite ionospheric conductivity, we use the approach of the paper (Hameiri and Kivelson, 1991) and its further modification (Hameiri, 1999) in application to ballooning modes.

2. Initial equations

As it was demonstrated earlier (Cheremnykh and Parnowski, 2004) the ballooning perturbations are self-generated MHD perturbations of plasma in the inner magnetosphere of the Earth. They are described by the following set of equations

$$\Omega^2\xi + \frac{a(\theta)}{\cos^{13}\theta}\frac{\partial}{\partial\theta}\left[\frac{1}{a(\theta)\cos\theta}\frac{\partial\xi}{\partial\theta}\right] + \frac{4}{a(\theta)\cos^4\theta}\left(T_0 + \frac{\alpha\beta}{\gamma}\xi\right) = 0, \tag{1}$$

$$\Omega^2\eta + \frac{1}{\cos^{13}\theta}\frac{\partial}{\partial\theta}\left[\frac{1}{\cos\theta}\frac{\partial\eta}{\partial\theta}\right] = 0, \tag{2}$$

$$\Omega^2\tau + \frac{1}{\cos^7\theta}\frac{\partial T_0}{\partial\theta} = 0, \tag{3}$$

where

$$T_0 = \beta\left[\frac{1}{\cos^7\theta}\frac{\partial}{\partial\theta}\left(\frac{\cos^{12}\theta}{a(\theta)}\tau\right) - \frac{4\cos^2\theta}{a^2(\theta)}\xi\right],$$
$$a(\theta) = 1 + 3\sin^2\theta, \quad \alpha = -\frac{L}{p}\frac{dp}{dL}, \quad \beta = \frac{\gamma p}{B_0^2}, \quad \gamma = \frac{5}{3}.$$

Here, L is McIlwain parameter, B_0 is the value of magnetic field on the magnetic equator, θ is the poloidal angle counted from the magnetic equator (geomagnetic latitude), $\Omega = \omega/\omega_A$, $\omega_A = B_0/L\sqrt{\rho}$ is the Alfvén frequency, all other notations are conventional. Eqs. (1) and (3) are coupled with each other and describe the interaction of poloidal Alfvén and slow magnetosonic modes. Eq. (2) describes toroidal Alfvén modes.

When deriving Eqs. (1)–(3), we neglected the deviations of geomagnetic field from the dipole one

$$\vec{\mathbf{B}} = \nabla\Psi \times \nabla\varphi, \tag{4}$$

where Ψ is the poloidal magnetic flux, φ is the toroidal angle (geomagnetic longitude). This is true at the considered finite values of pressure ($\beta < 1$). The magnetospheric plasma equilibrium is considered to be provided by the toroidal current at isotropic pressure with no convection. The plasma displacement vector $\vec{\xi}$ is represented in the following form:

$$\vec{\xi} = \xi\frac{\nabla\Psi}{|\nabla\Psi|^2} + \eta\frac{\vec{\mathbf{B}}\times\nabla\Psi}{|\vec{\mathbf{B}}|^2} + \tau\frac{\vec{\mathbf{B}}}{|\vec{\mathbf{B}}|^2}$$

provided the perturbation magnitudes ξ, η and τ satisfy the ballooning approximation

$$\frac{|\nabla\Psi\cdot\nabla X|}{|\nabla\Psi|}, \frac{|\vec{\mathbf{B}}\times\nabla\Psi\cdot\nabla X|}{|\vec{\mathbf{B}}||\nabla\Psi|} >> \frac{|X|}{b}, \frac{|\vec{\mathbf{B}}\cdot\nabla X|}{|\vec{\mathbf{B}}|}. \tag{5}$$

Here, b is the characteristic spatial scale of equilibrium values, and X stands for each one of ξ, η or τ.

The field lines of the geomagnetic field intersect the ionosphere's surface, causing electrodynamical interaction of magnetospheric and ionospheric plasmas. This leads us to the necessity of taking into account the influence of electrodynamical properties of the ionospheric plasma, and in the first place, its conductivity on the processes of generation of magnetospheric MHD perturbations. We will consider this influence further through the boundary conditions for the equations of small oscillations of the magnetospheric plasma. To obtain these boundary conditions, we will take into account the existence of two near-Earth layers: the insulating atmosphere and the partially ionized ionosphere.

3. Boundary conditions

The characteristic spatial scale of the magnetosphere is significantly greater than that of the ionosphere. For this reason, we may consider the ionosphere as a thin rigid conductive layer. Speaking of rigidity, we mean that the perturbations do not propagate through the lower boundary of the ionosphere, i.e., a following boundary condition holds

$$\vec{\xi}\cdot\vec{\mathbf{n}}\Big|_{\mathrm{b}} = 0. \tag{6}$$

Here, $\vec{\mathbf{n}}$ is an outward unit normal to the ionospheric layer; subscript b denotes the lower boundary of the ionosphere. Condition (6) excludes vertical displacements of plasma and is true due to large neutral population in the ionosphere and the presence of insulating atmosphere.

For formulation of the second boundary condition, we use the assumption of electrical charge conservation in a perturbed plasma medium. For the considered problem, it means that the perturbed current flowing out from the magnetospheric plasma serves as a source of currents flowing in the ionospheric layer. The boundary condition in this case has a well-known form (Hasegawa and Sato, 1989)

$$\vec{\mathbf{J}}_M\cdot\vec{\mathbf{n}}|_{\mathrm{b}} = \nabla_S\left(\overleftrightarrow{\mathbf{\Sigma}}\cdot\vec{\mathbf{E}}_S\right)|_{\mathrm{b}}, \tag{7}$$

where $\vec{\mathbf{J}}_M$ is a perturbed magnetospheric current, flowing to the ionospheric layer, $\overleftrightarrow{\mathbf{\Sigma}}$ is the tensor of integral conductivity of the ionospheric layer, subscript S denotes tangential components of the vector along the