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Low scale Higgs inflation with Gauss–Bonnet coupling

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ABSTRACT

Recent LHC data provides precise values of coupling constants of the Higgs field, however, these measurements do not determine its coupling with gravity. We explore this freedom to see whether Higgs field non-minimally coupled to Gauss–Bonnet term in 4-dimensions can lead to inflation generating the observed density fluctuations. We obtain analytical solution for this model and that the exit of inflation (with a finite number of e-folding) demands that the energy scale of inflation is close to Electro-weak scale. We compare the scalar and tensor power spectrum of our model with PLANCK data and discuss its implications.

1. Introduction

Cosmological Inflation [1–5] provides a causal mechanism to generate the primordial density perturbations that are responsible for the anisotropies in the cosmic microwave background (CMB) and the formation of the large scale structure (LSS). CMB and LSS data have been used to constrain the parameters of the inflationary model. In the case of canonical scalar field, the CMB and LSS data provide constraints on the height and the slope of the potential ref [6–8].

The fact that the temperature fluctuations of the CMB is close to scale-invariance is a highly demanding requirement for inflation model building ref [9–12] than providing approximately 60 e-foldings of inflation needed to solve the various initial conditions problems. More specifically, the near scale-invariance imposes a condition that the canonical scalar field potential should be almost flat — almost like cosmological constant — so that the quantum fluctuations that exit the horizon during inflation is nearly scale-invariant. While these flat potentials are phenomenologically successful, however, in the standard model of particle physics there is no candidate with such flat potentials that could sustain inflation [9–12]. For instance, the renormalizability of the Higgs field in 4-dimensions puts a constraint on the scalar field potential ($V(\phi) = m^2\phi^2 + \lambda\phi^4$, where m is the mass and λ is the coupling parameter), however, inflationary models require potentials of the form $V(\phi) = \sum_{n=0}^{N} c_{2n}\phi^{2n}$ where c_{2n}'s are real numbers and $N > 2$.

To achieve the flatness of the potential, inflationary models using the standard model Higgs field as the inflaton, couples Higgs field non-minimally with gravity [13–18]. In Higgs Inflation [13–15,19–25], the flatness of the Higgs potential is achieved by large non-minimal coupling of the Higgs field to the Ricci scalar ($\sim \xi R\phi^2$ where ξ is the coupling constant and R is the Ricci scalar) i. e. $\xi \sim 10^4$.

One of the main assumptions in the above models of Higgs inflation is that the standard model physics remains to hold until Planck energy. Which may be consistent with the current LHC measurements — since there are no evidence of new physics so far (e.g., supersymmetry or extra dimension(s), etc.) ref [26–28] — however, it also points to the fact that λ can be negative at high energies [29–35]. But a non-minimal Higgs Ricci coupling may prevent this up to inflationary scale [20,23].

In this work, we ask the following question: Can Higgs field drive inflation without invoking any new Physics in the particle physics sector with exit at low-energies, order of 100 GeV to 1000 GeV? While the LHC measurements determine the coupling constants of the Higgs field precisely, it does not determine its coupling with Gravity. We use this freedom and assume that the Higgs field couples with the Gauss-Bonnet Gravity, instead of Ricci Scalar.

Gauss-Bonnet Gravity is a part of the general extension of Gravity theories referred to as Lovelock theories of gravity [36]. One important feature of Lovelock theories, as against the $f(R)$ gravity theories, is that the gravity equations of motion remain second order (and quasilinear in second order). They provide a natural arena for understanding many deep features of gravity and recently they have been a subject of study. (For a recent review see [37].) Some higher dimensional Lovelock theories arise also as a weak field limit of string theory [38,39]. While a pure Gauss-Bonnet term is non-dynamical in 4-dimensions — topologically invariant in 4-dimensions — non-minimal coupling with the Higgs field makes it dynamical.

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Since Gauss-Bonnet term is higher-derivative term, it may be natural to expect that non-minimal coupling of the scalar field may only lead to modifications at high-energies. However, in this work, we show explicitly that the non-minimal coupling of the Higgs scalar lead to exit of inflation at low-energies i.e. close to Electroweak scale. This an unique feature of our model. We also explicitly compute the power-spectrum and show that it is consistent with the recent PLANCK data [8]. There has been recent interest in coupling scalar field with Gauss-Bonnet gravity (see, for instance, [18,40]). Our analysis differs from their analyses: In Ref. [40], authors have assumed that Einstein–Hilbert term is irrelevant and, hence, have ignored the linear term. In Ref. [18], the authors have coupled the scalar field to both the Ricci and Gauss-Bonnet gravity. Their analysis is based on slow-roll and makes predictions similar to [15]. It is important to note that they found the Gauss-Bonnet term to be significant only at late times where as the Higgs-Ricci coupling dominating the initial epoch and was responsible for the spectrum. As mentioned earlier, our model couples the Higgs scalar with Gauss-Bonnet gravity leading to a dynamical model of inflation.

The paper is organized as follows: In the next section, to get the physical picture of the dynamical equations, we obtain exact generalized power-law inflation for our model. We show that the generalized power-law solution exists only when the mass of the scalar field is identically zero. In Section 3, we show that the Higgs potential leads to dynamical model of inflation where the exit occurs close to the electro-weak scale. We show that the non-zero Higgs mass lead to the exit. In Section 4, we compute the power-spectrum of our model and compare the results with the recent PLANCK data. We discuss the key results and possible implications of our model in Section 5.

In this work, we consider $(-,+,+,+)$ metric signature. We use natural units $c=\hbar=1$, $\kappa = 1/M_{\rm Pl}^2$, and $M_{\rm Pl}^2 = \frac{\hbar c}{8\pi G}$ is the reduced Planck mass. We denote dot as derivative with respect to cosmic time t and $H(t) \equiv \dot{a}(t)/a(t)$.

2. Generalized power-law inflation

Consider the following action where the scalar field ϕ is non-minimally coupled of Gauss–Bonnet ($\mathcal{L}_{\rm GB}$) term:

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa} + f(\phi)\mathcal{L}_{\rm GB} - \frac{1}{2}\nabla_a\phi\nabla^a\phi - V(\phi)\right], \tag{1}$$

where R is the Ricci scalar, $V(\phi)$ is the scalar field potential, $f(\phi)$ is the coupling parameter and

$$\mathcal{L}_{GB} = R^2 - 4R^{ab}R_{ab} + R^{abcd}R_{abcd} \tag{2}$$

Varying the action (1) w.r.t the field ϕ and the metric leads to the following equations of motion [41]:

$$\Box\phi + \mathcal{L}_{GB}\, f_{,\phi}(\phi) - V_{,\phi}(\phi) = 0 \tag{3}$$

$$\begin{aligned}\frac{1}{\kappa}G_{pq} = \Big(&8G_{pq}g^{ab}\nabla_{ab}f(\phi) + 4R\nabla_{pq}f(\phi) - 8R^a_p\nabla_{aq}f(\phi)\\ &- 8R^a_q\nabla_{ap}f(\phi)8\nabla_{ab}f(\phi)R^{ab}g_{pq} - 8\nabla_{ab}f(\phi)R_{p\ q}^{\ a\ b}\\ &+ \nabla_p\phi\nabla_q\phi - g_{pq}\Big(\frac{1}{2}g^{ab}\nabla_a\phi\nabla_b\phi + V(\phi)\Big)\Big)\end{aligned} \tag{4}$$

It must be noted that the field equations being second order implies that this model doesn't have the problem of unitarity.

In this section, we are interested in obtaining exact solution for the above set of equations of motion for a spatially flat Friedmann–Robertson–Walker (FRW) background

$$ds^2 = -dt^2 + a(t)^2\left(dx^2 + dy^2 + dz^2\right) \tag{5}$$

where $a(t)$ is the scale factor. The equation of the field $\phi(t)$ and the scale factor $a(t)$ follows from Eqs. (3) and (4), respectively

$$-24H^2\frac{\ddot{a}}{a}f_{,\phi}(\phi) + \ddot{\phi} + V_{,\phi}(\phi) + 3H\dot{\phi} = 0 \tag{6a}$$

$$-3H^2\left(\frac{1}{\kappa} + 8H\dot{f}(\phi)\right) + \frac{1}{2}\dot{\phi}^2 + V(\phi) = 0 \tag{6b}$$

$$-H^2\left(\frac{1}{\kappa} + 8\ddot{f}(\phi)\right) - \frac{2\ddot{a}}{a}\left(\frac{1}{\kappa} + 8H\dot{f}(\phi)\right) + V(\phi) - \frac{1}{2}\dot{\phi}^2 = 0 \tag{6c}$$

It is important to note that the above differential equations are quasilinear i.e. they are linear with respect to all the highest order derivatives of $a(t)$ and $\phi(t)$. Rewriting Eqs. (6a) and (6b), we get

$$\begin{aligned}&-2H^2 + \kappa\dot{\phi}^2 - 24\,\kappa\, H^3\dot{f}(\phi) + 2\frac{\ddot{a}}{a} + 16\,\kappa H\frac{\ddot{a}}{a}\dot{f}(\phi)\\ &+ 8\kappa H^2\ddot{f}(\phi) = 0\end{aligned} \tag{7}$$

In the rest of this section, we consider the solution of (6) for the following ansatz

$$a(t) = a_0\left(\frac{t}{t_0} + \Upsilon\right)^p \ \text{ and } \ \phi = \phi_0\left(\frac{t}{t_0} + \Upsilon\right)^n \tag{8}$$

where $p > 1$ is a constant; n is a constant; a_0, t_0 are arbitrary constants whose values will not appear in any physical measured quantities and Υ is given by

$$\Upsilon = \left(\frac{\phi(t_0)}{\phi_0}\right)^{1/n} - 1\,. \tag{9}$$

Usually in cosmology, power-law inflation is given by $a(t) \propto t^p$. Ansatz (8) is a generalization. For real integer p, we have

$$a(t) = a_p t^p + a_{p-1}t^{p-1} + a_{p-2}t^{p-2} + \cdots + a_0$$

where in our case the coefficients $a_p, a_{p-1}, \cdots a_0$ are related. Since, $a(t)$ is a series, ϕ should also be a series like

$$\phi(t) = \phi_n t^n + \phi_{n-1}t^{n-1} + \phi_{n-2}t^{n-2} + \cdots + \phi_0$$

where, again, all the coefficients $\phi_n, \phi_{n-1}, \cdots \phi_0$. At $t = t_0$, $\phi(t_0) \neq \phi_0$ and ϕ_0 is an independent parameter. We refer to the above ansatz (8) for the scale factor as generalized power-law inflation.

Substituting the above ansatz (8) in Eq. (6), we get the following *exact relations*

$$V(\phi) = \tilde{\lambda}_1\,\phi^{-\frac{2}{n}} + \tilde{\lambda}_2\,\phi^{\frac{2(n-1)}{n}} + \tilde{\lambda}_3\,\phi^{\frac{p-1}{n}} \tag{10a}$$

$$f(\phi) = \tilde{\alpha}_1\,\phi^{\frac{2}{n}} + \tilde{\alpha}_2\,\phi^{\frac{2(n+1)}{n}} + \tilde{\alpha}_3\,\phi^{\frac{3+p}{n}} \tag{10b}$$

where

$$\tilde{\lambda}_1 = \frac{3(p-1)p^2}{\kappa(p+1)}\left(\frac{\phi_0^{1/n}}{t_0}\right)^2 \quad \tilde{\lambda}_2 = \frac{(5n^2p - n^2 + 2n^3)}{2(1-2n+p)}\left(\frac{\phi_0^{1/n}}{t_0}\right)^2$$

$$\tilde{\lambda}_3 = 24p^3C\left(\frac{\phi_0^{1/n}}{t_0}\right)^{1-p} \quad \tilde{\alpha}_1 = \frac{-1}{8\kappa\, p(1+p)}\left(\frac{\phi_0^{1/n}}{t_0}\right)^{-2}$$

$$\tilde{\alpha}_2 = \frac{n^2}{16p^2(1+n)(1-2n+p)}\left(\frac{\phi_0^{1/n}}{t_0}\right)^{-2}$$

$$\tilde{\alpha}_3 = \frac{C}{p+3}\left(\frac{\phi_0^{1/n}}{t_0}\right)^{-(p+3)} \tag{11}$$

and C is the constant of integration.

The following points are important to note regarding the above generalized power-law solution: (i) The ansatz (8) is the most general power-law *exact* solution satisfying Eqs. (6) for the potential and coupling (10) and *does not* depend on the constant of integration C. (ii) The above solutions are valid for any $p > 1$ and n. Imposing the condition that the potential be non-negative leads to $n > -2$ and $C \geq 0$. (iii) The coefficient of the first term in RHS of (10a) dominates the coefficients of the other two terms. Similarly,