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Maximum entropy analysis of cosmic ray composition

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ABSTRACT

We focus on the primary composition of cosmic rays with the highest energies that cause extensive air showers in the Earth's atmosphere. A way of examining the two lowest order moments of the sample distribution of the depth of shower maximum is presented. The aim is to show that useful information about the composition of the primary beam can be inferred with limited knowledge we have about processes underlying these observations. In order to describe how the moments of the depth of shower maximum depend on the type of primary particles and their energies, we utilize a superposition model. Using the principle of maximum entropy, we are able to determine what trends in the primary composition are consistent with the input data, while relying on a limited amount of information from shower physics. Some capabilities and limitations of the proposed method are discussed. In order to achieve a realistic description of the primary mass composition, we pay special attention to the choice of the parameters of the superposition model. We present two examples that demonstrate what consequences can be drawn for energy dependent changes in the primary composition.

1. Introduction

The mass composition of cosmic rays (CR) is an important issue in astroparticle physics research. The energy dependence of the primary mass distribution can provide useful information about ultra–high energy cosmic rays (UHECR) origin, their acceleration mechanisms and propagation through the galactic and extragalactic space. The mass observables can help to understand typical spectral features of UHECRs, the ankle observed at about 4 EeV and the steep flux suppression at energies above 30 EeV. In addition, the knowledge of the mass composition of UHECRs allows for an easier search for their sources or even investigation of basic characteristics of these sources.

In seeking for the masses of primary UHECR particles, the longitudinal development of extensive air showers (EAS) of secondary particles created in the Earth's atmosphere is usually examined. The penetration depth at which the CR shower reaches the maximum number of particles, $X_{\max}$, reflects the type of the primary particle causing this shower. The average depth of shower maximum for a set of CR showers detected at a given energy range, $\langle X_{\max}\rangle$, and its standard deviation, $\sigma_{\max} = \sigma(X_{\max})$, are then used to describe the main features of the primary mass composition. The quantitative interpretation of these data in terms of primary mass demands an accurate model of hadronic interactions. Usually, particles having a mass ranging from protons to iron nuclei are considered as responsible for the observed shower profiles of CR events. However, there is little information from the theory of what UHECR species are registered in current large CR detectors. Since the relevant phase space regions have not been explored in laboratory experiments, required interaction parameters are extrapolated from lower energy experiments, making the composition analysis uncertain.

Recent results from the Pierre Auger Observatory indicate a mixed CR composition with a transition from light to heavier primaries at the ankle region [1–4]. Measurements of $\langle X_{\max}\rangle$ show a flattening of the elongation rate near above 2 EeV. In addition, fluctuations of $X_{\max}$ expressed by the standard deviation $\sigma_{\max}$ were found to decrease from approximately 60 $\mathrm{g\,cm^{-2}}$ at 2 EeV to about 30 $\mathrm{g\,cm^{-2}}$ at 40 EeV. However, no such trends were observed by the HiRes and Telescope Array experiments. Their analyses prefer light primaries at the highest energies [5,6]. But it is not excluded that the observed inconsistencies may be due to the fact that detector effects were not fully eliminated. All the current experiments agree in suggesting a light CR composition below 2 EeV at the level of their systematic uncertainties [1–7].

Problems related to the mass composition of UHECRs have been widely communicated in the literature, see e.g. Refs. [8,9] for recent reviews. The energy dependencies of the average logarithmic

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mass and of its standard deviation, as measured by the Pierre Auger Observatory, have been recently examined under the assumption of selected models of hadronic interactions [4,10–12]. Other methods based on a given parametrization of the distribution of the depth of shower maximum for the study of the mass composition of UHECRs have been introduced, among others see e.g. Refs. [13,14]. The need of the muon number measurement is often emphasized for estimating the spread of masses in the beam of primary UHECR species [15–19]. However, recently analyzed data from the Pierre Auger Observatory indicates that the muonic component of air showers is not well described by the current models of hadronic interactions used for EAS simulations [20,21]. Also, different statistical tools have been used to obtain information about the primary mass composition on an event–by–event basis, see e.g. Refs. [22–24]. Finally, it is worth noting that the knowledge of the chemical composition of UHECRs was shown to play a crucial role in studies trying to describe the anisotropy signal and, consequently, to estimate properties of CR sources [25–29].

In case of experiments that measure the depths of shower maximum, the distribution of the mass of primary particles can be inferred only with the help of sets of simulated reference showers. However, since the shower properties are not yet fully understood, currently available models of hadronic interactions provide different solutions to the composition problem, for the recent analysis of the Auger data see Ref. [12]. Dealing with the mean and variance of the depth of shower maximum, mass observables of primary particles are usually estimated and, eventually, their relationship is examined using hadronic interaction models [8–10]. The power of this combined analysis has been repeatedly emphasized. Nevertheless, the predictions of existing models are different and in some cases even indicate possible inconsistencies in the modeling of hadronic interactions [4,10].

Inspired by these findings, we examine what can actually be obtained using just a set of the two lowest order $X_{\max}$ moments. We address the issue of how to assess what trends in the primary composition are most strongly supported by this data if only a limited knowledge of EAS properties is available. The proposed method and the interpretation of its results is quite distinct from and independent of other more conventional procedures used in composition studies. In particular, our inference procedure is designed to exploit incomplete information about investigated phenomena and provides their probabilistic interpretation.

With the aim to deduce relative occupancy of primary particles, we relate shower observables to average masses of incident primaries using an air shower model. We intentionally made an attempt to account for the basic properties of the longitudinal EAS development independently of the assumptions about detailed features of hadronic interactions. Instead, we used the fact that the current data and its subsequent analysis, when faced with air shower simulations, are not able to undermine the validity of a simple superposition ansatz [8]. This choice allows us to classify obtained solutions in the space of physically reasonable parameters. Moreover, it enables us to assess the properties of different models of hadronic interactions. Finally, when we present the resultant primary composition, existing knowledge about EAS physics is considered. In our treatment, the superposition model was supplemented by simple considerations providing us other mass dependent terms relevant for EAS physics [8]. As a result, the two lowest order $X_{\max}$ moments were parametrized in a similar manner as originally suggested in Refs. [30,31].

We focused on how to gain credible information on the primary mass composition that takes account of our incomplete knowledge of the underlying processes leading to the observation of the two lowest order $X_{\max}$ moments. For this purpose, we adopted the principle of maximum entropy [32–35]. This criterion, without any other assumptions about $X_{\max}$ data, allows us to choose a unique well–behaved solution among various options how to combine primary components so as to obtain the two lowest order sample moments.

The method of maximum entropy relies on the properties of entropy as a measure of uncertainty. It sets the task to return a maximally noncommittal distribution of a quantity that is constrained by information obtained in experiment. It is worth stressing that such a solution does not necessarily provide an unambiguous description of the process that generates the observed data. Instead, this method provides us with the probability distribution of the underlying quantity which is most strongly supported by the facts while using as little additional information as possible in order to avoid unintentionally assuming more than is really known. This scheme is not only backed by a compelling statistical motivation, but also fairly simple to implement, yet sufficiently general. It is widely used in many branches of science, for recent review of its basic ingredients, aspects and applications see e.g. Ref. [35].

In the context of composition studies, the proposed method treats the two lowest order $X_{\max}$ moments, and possibly other average shower observables, on an equal footing. Having these moments, the probabilities are uniquely assigned to selected primary particles that are assumed to cause observed air showers. For this, we need a specific shower model that converts shower observables into the mass number space. The resultant distribution of the incident particles is then obtained from the available data without any further assumptions about the properties of this data. Such a solution enables us to draw minimally biased conclusions about the composition of the beam of primary particles within the framework of a chosen shower model. More importantly, we can find a set of acceptable solutions with maximum entropy in the parameter space of the shower model and check whether the available models of hadronic interactions can provide such solutions. The analogous interpretation of mass composition measurements does not seem to have been previously documented.

The structure of the paper is as follows. In Section 2, the air shower model is introduced supplemented by Appendix A. Particular attention is paid to the choice of model parameters. The original contribution of our study, the inference procedure for the composition determination, is described in Section 3. In this section, we present a way to use the partition method and point out the probabilistic interpretation of its output. The essential features of the underlying principle of maximum entropy are summarized in Appendix B. Examples are presented and discussed in Section 4. The paper is concluded in Section 5.

2. Air shower model

Let us assume that a depth of shower maximum $X_{\max}$ is observed when a UHECR particle with mass A and energy E hits the upper part of the Earth's atmosphere. We treat the former two quantities as dependent random variables, $X_{\max} = X_{\max}(A)$. The primary energy is considered to be a known parameter. For the longitudinal shower development we utilize the superposition model in which a primary nucleus is regarded as a superposition of A nucleons of energy E/A. We assume that the mean depth of shower maximum of a set of showers caused by the same primaries is a linear function of the decimal logarithm of their energies per nucleon [8]

$$\langle X_{\max} \mid A \rangle = C + D\,\mathrm{Log}\left(\frac{E}{E_0 A}\right). \tag{1}$$

Here, $C = \langle x_{\max} \rangle (E_0)$ denotes the mean depth of shower maximum for protons with a reference energy of E_0, $D = \frac{\mathrm{d}\langle x_{\max}\rangle}{\mathrm{dLog}E}$ is the proton elongation rate and the proton mean depth of shower maximum is denoted by $\langle x_{\max} \rangle = \langle x_{\max} \rangle (E) = \langle X_{\max} \mid A = 1 \rangle$. The model