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Significance in gamma ray astronomy with systematic errors

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ABSTRACT

The influence of systematic errors on the calculation of the statistical significance of a γ-ray signal with the frequently invoked Li and Ma method is investigated. A simple criterion is derived to decide whether the Li and Ma method can be applied in the presence of systematic errors. An alternative method is discussed for cases where systematic errors are too large for the application of the original Li and Ma method. This alternative method reduces to the Li and Ma method when systematic errors are negligible. Finally, it is shown that the consideration of systematic errors will be important in many analyses of data from the planned Cherenkov Telescope Array.

1. Introduction

The calculation of the statistical significance of a measured signal is a central part of a data analysis in Very High Energy (VHE) γ-ray astronomy. In a typical analysis, the number of events, N_{ON}, that are detected in a signal region is compared to the number of events, αN_{OFF}, that are expected in the signal region. The expected number of events in the signal region results from a measurement of N_{OFF} events in a dedicated background region. Different methods (see [1]) exist to construct the background region for a given signal region such that the event acceptances in the signal and the background region are equal. If the event acceptances in the signal and the background region are equal, the normalization factor α is equal to the ratio of the exposures in the signal and the background region.

The statistical significance of a γ-ray signal, $\Delta = N_{ON} - \alpha N_{OFF}$, is derived in [2] to be

$$S_{LiMa}\ (N_{ON}, N_{OFF}, \alpha) = \text{sign}\ (\Delta) \times \sqrt{-2\left(N_{ON}\ \ln\left(\frac{\overline{N_{ON}}}{N_{ON}}\right) + N_{OFF}\ \ln\left(\frac{\overline{N_{OFF}}}{N_{OFF}}\right)\right)} \quad (1)$$

where $\overline{N_{ON}} = \alpha\overline{N_{OFF}}$ and

$$\overline{N_{OFF}} = \overline{N_{OFF}}(\alpha) = \frac{N_{ON} + N_{OFF}}{1+\alpha}\ . \quad (2)$$

The normalization factor α is taken to have a negligible error, σ_α, in the derivation of Eq. (1) in [2].

The order of magnitude of the statistical error on the γ-ray signal due to Poisson fluctuations of N_{ON} and N_{OFF} can be estimated to be $\Delta_{Background} \approx \sqrt{N_{ON} + \alpha^2 N_{OFF}}$. When no signal is present, it holds $N_{ON} \approx \alpha N_{OFF}$ which leads to $\Delta_{Background} \approx \sqrt{\alpha(\alpha+1)N_{OFF}}$. A similar back of the envelope estimation for the error on the γ-ray signal, propagated from an error on the normalization factor, leads to $\Delta_\alpha \approx \sigma_\alpha N_{OFF}$.

The application of S_{LiMa} to calculate the statistical significance of a γ-ray event signal is justified if $\Delta_\alpha \ll \Delta_{Background}$. Using the back of the envelope estimation for the case where no signal events are measured in the signal region, the condition $\Delta_\alpha \ll \Delta_{Background}$ translates into the condition

$$\frac{\sigma_\alpha}{\alpha} \ll \sqrt{\frac{1+\alpha}{\alpha}\frac{1}{N_{OFF}}} \quad (3)$$

for the relative error on the normalization factor.

The following discussion focuses on data acquired with imaging atmospheric Cherenkov telescopes (IACTs). However, S_{LiMa} is also frequently applied in analyses of data from ground based water Cherenkov telescopes (see e.g. [3,4]) and extensive air shower arrays (see e.g. [5]).

The High Energy Stereoscopic System (H.E.S.S.) is an array of IACTs that operates in the Namibian Khomas Highland since 2003. Observations of the Crab nebula with H.E.S.S. result in the detection of approximately 100 background events that pass standard Hillas γ-ray event selection criteria per 30 min observation time for a normalization factor of $\alpha = 0.2$ (see [6] for details). Similar background event rates hold for analyses of data from

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observations of point like γ-ray sources with other current generation IACT arrays such as MAGIC [7] and VERITAS [8]. According to Eq. (3), the relative error on the normalization factor must be much smaller than $\sigma_\alpha/\alpha \approx 25\%$ when S_{LiMa} is used to calculate the significance of a γ-ray signal with $N_{\mathrm{OFF}} \approx 100$ and $\alpha = 0.2$. More precisely, it will be shown later that in this case σ_α/α must be known to about 3% to justify the application of S_{LiMa}.

For the planned Cherenkov Telescope Array (CTA, see [9]), the increase in the number of telescopes will (compared to current generation IACTs) lead to a factor of 10 larger effective area. An increase of the background event rate for CTA of a similar factor of 10 (again compared to current generation IACT arrays) is expected from the enlarged effective area (see [10]). Consequently, approximately 10^3 background events per 30 min observation time are expected in a typical point source analysis of CTA data with $\alpha = 0.2$. In this case, it is estimated with Eq. (3) that the relative error on the normalization factor must be much smaller than 8%. Otherwise, an error on the normalization factor must be considered. Again more precisely, it will be shown later that the relative error on the normalization factor must be known to about one order of magnitude better than 8% for the application of S_{LiMa} in this situation.

In analyses of extended γ-ray sources (e.g. Supernova Remnants, [11]) or galactic dark matter searches (e.g. [12]), the increased size of the signal region leads easily to background event rates which are an order of magnitude larger than for point source analyses. It is obvious that in those cases, the normalization factor α must be known with an even better precision than in point source analyses.

It is arguable whether the normalization factor is in general known with the precision that is required for the application of S_{LiMa}. This holds in particular for analysis of data from the planned CTA experiment.

This paper extends the method for the calculation of the statistical significance of a γ-ray signal first proposed in [2] to include an error on the normalization factor. In addition to other authors, who discussed the same problem (e.g. [13,14]), a simple expression for the calculation of the significance of a measured γ-ray signal is derived. Moreover, the criterion given by Eq. (3) to decide whether a given error on the normalization factor must be considered in the calculation of the statistical significance of a γ-ray signal is tested in Monte Carlo simulations.

The structure of the paper is as follows: The effect of the neglection of a systematic error on the normalization factor on S_{LiMa} is quantitatively discussed in Section 2. A modified expression for the significance calculation, which considers an error on the normalization factor, is derived in Section 3.

Section 4 discusses more specifically the influence of systematic errors on the calculation of statistical significances in VHE γ-ray astronomy with IACTs like H.E.S.S. or CTA.

2. The Li and Ma significance with a Gaussian distributed normalization factor

Fig. 1 shows the distribution of the significance as calculated with S_{LiMa} when $N_{\mathrm{ON}} = \mathrm{Pois}(\alpha b)$ and $N_{\mathrm{OFF}} = \mathrm{Pois}(b)$ are independently Poisson distributed with $b = 500$ events. No signal events are simulated and it is expected that the Gaussian fit of the distribution of the significances results in a distribution which is compatible with being a standard normal. The distribution of the significance is shown in Fig. 1 for a fixed normalization factor, $\alpha = 0.2$. Additionally shown is the distribution of S_{LiMa} for two Gaussian distributed normalization factors. The mean of the assumed distributions of the normalization factor is $\alpha = 0.2$ in both cases. Respectively, the relative widths of the Gaussian distributions for the normalization factor are $\sigma_\alpha/\alpha = 8\%$ and $\sigma_\alpha/\alpha = 15\%$. The assumption of a Gaussian distributed normalization factor is here and in the following restricted to small relative errors on the normalization factor ($\sigma_\alpha/\alpha \lesssim 15\%$). For large relative errors on the normalization factor, the distribution cannot in general be assumed to be Gaussian, e.g. because the normalization factor must be larger than zero for physical reasons. However, the assumption of small relative errors on the normalization factor is reasonable because large relative errors can easily be identified and corrected for in analyses.

The mean of all fitted significance distributions that are shown in Fig. 1 is compatible with zero. However, the width of the Gaussian fit to the significance distribution is only compatible with being one when the error on the normalization factor vanishes. For non-zero relative errors on the normalization factor, the width of the Gaussian fit to the significance distribution increases with the relative error on the normalization factor. In other words, the absolute value of the significance is overestimated by S_{LiMa} when the relative error on the normalization factor is not vanishing.

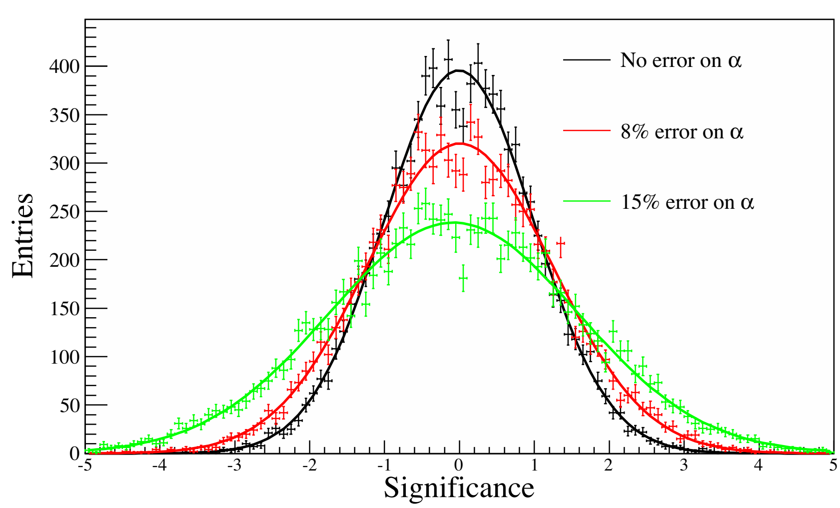

Fig. 1. Distribution of S_{LiMa} (Eq. (1)) when $N_{\mathrm{ON}} = \mathrm{Pois}(\alpha b)$ and $N_{\mathrm{OFF}} = \mathrm{Pois}(b)$ are independently Poisson distributed with $b = 500$ events. The normalization factor, α, is distributed like a Gaussian with mean 0.2 and relative width according to the legend. The width of the Gaussian fit to the significance distribution is compatible with being one if the error on the normalization factor is vanishing. However, the width of the Gaussian fit to the significance distribution increases with the relative error on the normalization factor.