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Modified Rindler acceleration as a nonlinear electromagnetic effect

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ABSTRACT

The model proposed originally by Mannheim and Kazanas for fitting the shapes of galactic rotation curves has recently been considered by Grumiller to describe gravity of a central object at large distances. Herein we employ the same geometry within the context of nonlinear electrodynamics (NED). Pure electrical NED model is shown to generate the novel Rindler acceleration term in the metric which explains anomalous behaviors of test particles/satellites. Remarkably a pure magnetic model of NED yields flat rotation curves that may account for the missing dark matter. Weak and strong energy conditions are satisfied in such models of NED.

1. Introduction

In Newton's theory of gravitation which reined for centuries, mass constituted the principal source for its potential. With the advent of general relativity, different sources were unified under spacetime texture such that the overall effective force used to matter. Thus, gravity/geometry can easily be attributed to non-mass originated sources equally well due to the manifestation of mass-energy equivalence. As a particular example we recall the Reissner-Nordström (RN) geometry of general relativity in which mass and charge coexist in making the geometry. Assuming that the source has negligible mass versus a significant charge the entire geometry can be attributed to the charge alone. In performing this process one should be cautious that no physical energy conditions are violated. Recent observations suggest that there are dark matter/energy that is associated with non-observable sources. As a result our detectable/observable matter falls rather short to account for the accelerated expansion of our universe. Before suggesting proposals for new forces/matter it is more logical to exhaust every kind of physical sources that we are at least familiar so that we know how to cope with. To explain the hierarchy of forces at small distances and close the gap of discrepancy between gravity and other fields, for instance, the idea of higher dimensions/branes was proposed [1,2]. Although no branes have been identified so far theoretical explanation such as dilution (weaking) of gravity among branes in higher dimensions remains consistently intact. As the gravity is tamed at UV scales by virtue of higher dimensions at the IR scales, or long distances does everything go perfect?. The recent proposal [3–6] that at large distances there is an additional parameter known as Rindler acceleration was rather unprecedented and the present paper is about the source of such a term.

We recall that in the near horizon limit, i.e. $r = 2m + x$ for $|x|^2 \ll 1$, a Schwarzschild black hole leads to the standard Rindler acceleration. Such an extraneous term must be purely general relativistic coupled with physical sources which lacks a Newtonian counterpart. Even in the Einstein–Maxwell version of general relativity with spherical symmetry such a term did not arise. The long range fields, i.e. gravitation and electromagnetism, manifest their inverse square law character so that asymptotically the spacetime becomes flat. Different sources such as dilatons, nonlinear electromagnetic fields and others admit non asymptotically flat solutions at large spatial distances. The difficulty with the new Rindler acceleration is that it violates both the Newtonian and Maxwellian limits: for large distances ($r \to \infty$) it becomes even more significant. In Newtonian terms the potential that gives inverse force law modifies into $\phi(r) = -\frac{m}{r} + ar$, where m is the central Newtonian mass and a is the novel Rindler acceleration under question. Unless the central object is supermassive and a is negligibly small it can be argued that for large r the new term dominates over the mass term. Further, the Rindler acceleration is not a universal constant as observationally it shows slight variations from Sun-Pioneer pair ($\sim 10^{-61}$ natural unit of acceleration which is equivalent to $10^{-10}\,\frac{m}{s^2}$ in physical units) to galaxy-Sun system $\left(\sim 10^{-62}\right)$ and others. We recall that such a linear dependence of potential on distance is encountered in parallel plates endowed

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with a uniform electric field in linear Maxwell electromagnetism (i.e. $V_0 = E_0 z, E_0 = \text{constant}$).

Gravity coupled with linear Maxwell electromagnetism in spherically symmetric geometry produces no such linear potential term either. For this reason we resort from the outset to nonlinear electrodynamics (NED) and prove a theorem to generate the new acceleration term. Truly it yields the required expression, however, in addition it gives as a by product an extra constant term in the metric which can be interpreted as a global monopole [7,8]. This amounts to further modification of the Newtonian potential by $\phi(r) = k - \frac{m}{r} + ar$, with the global monopole term $k = \text{constant}$. Our formalism suggests that both the Rindler acceleration (a) and global monopole (k) constants depend on the nonlinear electric charge of the heavenly object under consideration. That is, neither one is a fundamental constant of nature as both are derived from the charge. Interestingly the monopole term plays the similar role of a cosmological constant, i.e. a uniform electric field in the presence of NED-coupled gravity with nonisotropic difference. The upper bounds for both $|a|$ and $|k|$ have been tabulated for different planets. It is further shown that the monopole term is crucial for the weak and strong energy conditions to be satisfied. The spectrum of NED theories is very large and the problem is to find the proper Lagrangian that suits and serves for the purpose. Finally we observed that the Rindler acceleration does not account for the constant tangential velocity of circular orbits in the presence of mysterious dark matter. For this reason we have further modified the Rindler term in the metric function by $2ar \to 2ar_0 \ln r$ (with a and r_0 constant), which necessitates a new NED Lagrangian. For such a magnetic Lagrangian it is shown that the energy conditions are satisfied at the cost of a bounded universe. Further, the circular orbit around remote galaxies, has velocity $v = \sqrt{\frac{m}{r} + ar_0}$ which yields a better estimate between Newton and Rindler acceleration models toward accounts of dark matter.

2. The solutions

2.1. Pure electric case

Recently, Grumiller considered the Mannheim–Kazanas (MK) metric to describe gravity of a central mass at large distances which attracted interest due to its cosmological implications [5,6]. We must add that a linear term in the metric was first introduced in [3], and it was applied in earnest in fitting the shapes of galactic rotation curves by Mannheim (see [4], for a review). The novelty in this model is the inclusion of a term interpreted as Rindler acceleration. We wish to show in this paper that nonlinear electrodynamics (NED) may be responsible for the generation of such a term. Our starting point is the action

$$S = \int d^4x\sqrt{-g}[R - 2\Lambda + L(\mathcal{F})] \tag{1}$$

in which $R = $ Ricci scalar, $\Lambda = $ cosmological constant and $L(\mathcal{F})$ is the Lagrangian for the NED.

Before we choose the form of $L(\mathcal{F})$ we would like to add that $L(\mathcal{F})$ is not similar to the original BI Lagrangian. In Born–Infeld (BI) initial work the idea was the removal of the singularity at the origin. Following the classical charge with a finite size and a well defined charge distribution admitted what we call it BI Lagrangian. In what we introduce the singularity at the origin is not our worry any more and instead we are adjusting our Lagrangian to justify the behavior of the Galaxies at very large distance. Hence, the only constraint we impose on our Lagrangian is to satisfy the Maxwell equation with a single electric or magnetic fields. No need to mention that such an arbitrary Lagrangian may not give the Maxwell limit at large distance which otherwise expecting the Mannheim–Kazanas instead of Reissner-Nordström would be meaningless.

Our notation is such that $\mathcal{F} = F_{\mu\nu}F^{\mu\nu}$ represents the Maxwell invariant with the choice of Lagrangian

$$L(\mathcal{F}) = \frac{\alpha}{\sqrt{2\beta} - \sqrt{-\mathcal{F}}}. \tag{2}$$

Here $\alpha > 0$ is the coupling constant and $\beta > 0$ plays the role of the uniform background electric field as will be clarified in the sequel. Our original model Lagrangian (2) can be employed with the choice $\beta = 0$ as well. Note that $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the standard Maxwell field tensor and this Lagrangian will break the scale invariance, i.e. $x \to \lambda x, A_\mu \to \frac{1}{\lambda}A_\mu$, for $\lambda = \text{constant}$. We consider a static, spherically symmetric (SSS) spacetime described by the line element

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{3}$$

with the electric field ansatz.

The fact that the desired static spherically symmetric (SSS) line element (3) derives from the NED Lagrangian (2) through the action (1) can be formulated as a theorem [9].

Theorem 1. *Let our action be (1) with line element (3) and $L(F)$ be our pure electric NED Lagrangian described by the Maxwell 2−form*

$$\mathbf{F} = E(r)dt \wedge dr \tag{4}$$

satisfying the Maxwell's equation

$$d(^\star\mathbf{F}L_{\mathcal{F}}) = 0 \tag{5}$$

in which $^\star\mathbf{F}$ means dual of $\mathbf{F}$ and $L_{\mathcal{F}} = \frac{\partial L}{\partial \mathcal{F}}$. Then, the energy–momentum tensor satisfies the conditions $T_t^t = T_r^r$ and $T_\theta^\theta = T_\varphi^\varphi$ and Lagrangian $L(\mathcal{F})$ is related to the metric function $f(r)$ through

$$L = L_0 + 2\int \frac{1}{r^2}\left[r^2\left(\frac{f''}{2} + \frac{1-f}{r^2}\right)\right]' dr \tag{6}$$

where $L_0 = const.$ and a 'prime' implies $\frac{d}{dr}$.

Proof. Variation of the action (1) with respect to the metric tensor $g_{\mu\nu}$ yields the field equations in the form

$$G_\mu^\nu + \Lambda\delta_\mu^\nu = T_\mu^\nu \tag{7}$$

where G_μ^ν is the Einstein tensor and T_μ^ν is the energy–momentum tensor given by

$$T_\mu^\nu = \frac{1}{2}\left(L\delta_\mu^\nu - 4L_{\mathcal{F}}F_{\mu\lambda}F^{\nu\lambda}\right) \tag{8}$$

which admits $T_t^t = T_r^r = \frac{1}{2}L - \mathcal{F}L_{\mathcal{F}}$ and $T_\theta^\theta = T_\varphi^\varphi = \frac{1}{2}L$. From the line element (3) we find

$$G_t^t = G_r^r = \frac{rf' - 1 + f}{r^2} \tag{9}$$

and

$$G_\theta^\theta = G_\varphi^\varphi = \frac{rf'' + 2f'}{2r}. \tag{10}$$

The electric field 2−form has the dual given by

$$^\star\mathbf{F} = -E(r)r^2\sin\theta d\theta \wedge d\varphi \tag{11}$$

and the Maxwell's equation (5) implies that

$$E(r)r^2L_{\mathcal{F}} = const. = Q \tag{12}$$

in which Q is a charge related integration constant. Recall that, $L_{\mathcal{F}} = \frac{\partial L}{\partial \mathcal{F}} = \frac{L_E}{\frac{\partial \mathcal{F}}{\partial E}}$ and since $\mathcal{F} = -2E^2$, we have $L_{\mathcal{F}} = -\frac{L_E}{4E}$. Comparing this with Eq. (12) yields