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Astrophysical constraints on dark energy

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ABSTRACT

Dark energy (i.e., a cosmological constant) leads, in the Newtonian approximation, to a repulsive force which grows linearly with distance and which can have astrophysical consequences. For example, the dark energy force overcomes the gravitational attraction from an isolated object (e.g., dwarf galaxy) of mass $10^7 M_\odot$ at a distance of 23 kpc. Observable velocities of bound satellites (rotation curves) could be significantly affected, and therefore used to measure or constrain the dark energy density. Here, *isolated* means that the gravitational effect of large nearby galaxies (specifically, of their dark matter halos) is negligible; examples of isolated dwarf galaxies include Antlia or DDO 190.

1. Introduction

The discovery of dark energy, which accounts for the majority of the energy in the universe, is one of the most significant of the last 20 years. While the repulsive properties of dark energy are well known in the cosmological context, they have not been as thoroughly understood on shorter, astrophysical, length scales. Previous work has constrained the cosmological constant on solar-system scales [1], but its effects are obviously too small to be directly observed.

In what follows, we discuss the repulsive dark energy force and its astrophysical effects on galactic scales. Because this force grows linearly with distance, its effect is most likely to be significant for weakly-bound satellites with large orbits. To detect the effects of dark energy, we must first understand orbits resulting from ordinary gravitational dynamics, with potentials mainly determined by the distribution of dark matter. We find that observations of distant satellites of isolated dwarf galaxies could be used to detect the effects of dark energy. Here, isolated means sufficiently far from other sources of gravitational potential. When dwarf galaxy systems are not sufficiently isolated, the orbits of their satellites are subject to tidal forces from nearby large galaxies. These tidal forces can distort orbital shapes, and enforce an upper limit on orbital radii.

2. Newtonian gravity and cosmological constant

The Einstein equation with cosmological constant Λ is

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} + g_{\mu\nu} \Lambda. \tag{1}$$

Contracting both sides with $g^{\mu\nu}$, one gets $R = -8\pi G T - 4\Lambda$ where $T \equiv T^\mu_\mu$ is the trace of the matter (including dark matter) energy-momentum tensor. This can be substituted in the original equation to obtain

$$R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right) - g_{\mu\nu} \Lambda. \tag{2}$$

In the Newtonian limit, one can decompose the metric tensor as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $|h_{\mu\nu}| \ll 1$. Specifically, we are interested in the 00-component of the Einstein equation. We parameterize the 00th-component of the metric tensor as

$$g_{00} = 1 + 2\Phi, \tag{3}$$

where Φ is the Newtonian gravitational potential. To leading order, one can show that [2]

$$R_{00} \approx \frac{1}{2} \vec{\nabla}^2 g_{00} = \vec{\nabla}^2 \Phi. \tag{4}$$

In the inertial frame of a perfect fluid, its 4-velocity is given by $u_\mu = (1, \vec{0})$ and we have

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu} = \mathrm{diag}(\rho, p), \tag{5}$$

where ρ is the energy density and p is the pressure. For a Newtonian (non-relativistic) fluid, the pressure is negligible compared to the energy density, and hence $T \approx T_{00} = \rho$. As a result, in the Newtonian limit, the 00-component of the Einstein equation reduces to

$$\vec{\nabla}^2 \Phi = 4\pi G \rho - \Lambda, \tag{6}$$

which is just the modified Poisson equation for Newtonian gravity, including cosmological constant. This equation can also be derived from the Poisson equation of Newtonian gravity, $\vec{\nabla}^2 \Phi = 4\pi G (\rho + 3p)$, with source terms from matter and dark energy; $p \approx 0$ for non-relativistic matter, and $p = -\rho$ for a cosmological constant.

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Table 1
Galaxy masses (units of solar mass $M_\odot$) and the corresponding r_c.

Galaxy Mass	r_c
$10^6 M_\odot$	10.7 kpc
$10^7 M_\odot$	23.1 kpc
$10^8 M_\odot$	49.8 kpc
$10^9 M_\odot$	107 kpc
$10^{10} M_\odot$	231 kpc
$10^{11} M_\odot$	498 kpc
$10^{12} M_\odot$	1.07 Mpc
$10^{13} M_\odot$	2.31 Mpc
$10^{14} M_\odot$	4.98 Mpc

Assuming spherical symmetry, we have $\vec{\nabla}^2\,\Phi = \frac{1}{r^2}\,\frac{\partial}{\partial r}\left(r^2\,\frac{\partial \Phi}{\partial r}\right)$ and the Poisson equation is easily solved to obtain

$$\Phi = -\frac{GM}{r} - \frac{\Lambda}{6}\,r^2, \tag{7}$$

where M is the total mass enclosed by the volume $\frac{4}{3}\pi r^3$. The corresponding gravitational field strength is given by

$$\vec{g} = -\vec{\nabla}\,\Phi = \left(-\frac{GM}{r^2} + \frac{\Lambda}{3}\,r\right)\hat{r}. \tag{8}$$

Therefore, the cosmological constant leads to a repulsive force whose strength grows linearly with r.

One can also derive $\vec{g}$ by starting with the de Sitter-Schwarzschild metric [3]

$$ds^2 = \left(1 - \frac{2GM}{r} - \frac{\Lambda}{3}r^2\right)dt^2 - \left(1 - \frac{2GM}{r} - \frac{\Lambda}{3}r^2\right)^{-1} dr^2 - r^2 d\Omega^2 \tag{9}$$

which describes the spacetime outside a spherically symmetric mass distribution M in the presence of a cosmological constant Λ. One then obtains Eq. (7) and hence Eq. (8) by identifying Eq. (3) with the 00-component of the de Sitter-Schwarzschild metric.

3. Galaxies

The results obtained in previous section are relevant for galaxies. For instance, in the presence of the cosmological constant Λ, Eq. (8) describes the Newtonian gravitational field strength outside a galaxy with a spherically symmetric mass distribution M. From Eq. (8), it is clear that when r is sufficiently large, the repulsive dark force will dominate over the gravitational attraction. The critical value of r beyond which this happens is given by

$$r_c = \left(\frac{3\,G\,M}{\Lambda}\right)^{1/3} = \left(\frac{3\,M}{8\,\pi\,\rho_\Lambda}\right)^{1/3}, \tag{10}$$

where $\rho_\Lambda = \frac{\Lambda}{8\,\pi\,G} \approx (\,2.3 \times 10^{-3}\text{eV}\,)^4$ is the observed energy density of the cosmological constant. Table 1 displays galactic masses in units of solar mass $M_\odot$ and the corresponding r_c.

Typical galaxies, including our Milky Way, have total mass (including dark matter) $\gtrsim 10^{11-12}\,M_\odot$ and sizes ~ 50 kpc. According to Table 1, $r_c \gtrsim 500$ kpc for these galaxies, so the dark force is not likely to affect internal dynamics, but may impact galaxy–galaxy interactions [4], and limit the size of galaxy clusters ($\sim 10^{14}M_\odot$, size $\sim$ Mpc).

Some dwarf galaxies have total mass (including dark matter) $\sim 10^7 M_\odot$. These include Ursa Major II, Coma Berenices, Leo T, Leo IV, Canes Venatici I, Canes Venatici II, and Hercules (analyzed by [5]), and also Leo II [6] and Leo V [7]. The irregular galaxies Leo A [8], Antlia [9] and DDO 190 [10] also have masses around $10^7 M_\odot$.

For galaxies with mass $\sim 10^7 M_\odot$, we have $r_c \sim 23$ kpc. Thus, their galactic rotation curves could be affected by the dark force: rotational velocities of stars or gas clouds bound to these galaxies should be smaller than that predicted by ordinary Newtonian gravity. This in turn could provide a novel way to measure the cosmological constant in the future. Rotation curves for many galaxies have been measured to radii of ~ 30 kpc or more, and for some dwarf galaxies to ~ 10 kpc [11]. Low surface brightness (LSB) galaxies may also be worthy of investigation [12]. Some LSBs with total mass $\sim 10^{10} M_\odot$ have disks as large as 100 kpc.

The Navarro–Frenk–White (NFW) profile [13] is a commonly used parametrization of dark matter halo energy density:

$$\rho = \frac{\rho_0}{r/R_s\,(1 + r/R_s)^2}, \tag{11}$$

where ρ_0 is a characteristic halo density and R_s is the scale radius. These two quantities vary from galaxy to galaxy. While the detailed shape of the actual dark matter density may differ from the NFW profile, the asymptotic $1/r^3$ behavior is widely accepted. Our results below will not be sensitive to the density profile at small r.

Consider a dwarf galaxy (DG) and a larger galaxy (LG) (e.g., the Milky Way) whose centers of mass are separated by a distance R, and a satellite of the DG whose orbital radius is roughly r. If the distance R is sufficiently large, we can neglect the gravitational potential of the LG and treat the DG-satellite system as approximately isolated. In that case, the values in Table 1 provide a rough guide for distances r at which the dark energy force becomes significant. In the following section we will investigate to what extent measurement of satellite velocities can constrain the dark energy density around the DG.

But first let us examine in more detail under what circumstances we can neglect the gravitational effects from the (dark matter halos) of neighboring galaxies on the DG. We will assume an NFW profile for both the DG halo and the larger galactic halo. The distance R from the center of the DG to the center of the LG is generally not equal to the distance from the satellite to the center of the LG, which can vary from $(R - r)$ to $(R + r)$. Therefore, the gravitational pull exerted on the satellite by the LG is different from the pull on the DG, leading to a tidal effect. (See [14] for previous work regarding the tidal effects on orbiting satellites around their host galaxies.) This tidal effect is repulsive: it pulls apart the DG-satellite system. Perhaps surprisingly, for many DGs (i.e., near the Milky Way), the tidal effect is large enough to distort and even destabilize the satellite orbits.

Let the total mass of the DG enclosed within r be $M_{\rm DG}(r)$ and that of the LG enclosed within $R \pm r$ be $M_{\rm LG}(R \pm r)$. Then we have

$$M_{\rm DG}(r) = \int_0^r 4\,\pi\,r'^2\,\rho_{\rm DG}(r')\,dr', \tag{12}$$

$$M_{\rm LG}(R \pm r) = \int_0^{R\pm r} 4\,\pi\,r'^2\,\rho_{\rm LG}(r')\,dr'. \tag{13}$$

The circularity and stability of the satellite orbits can be guaranteed by requiring that the magnitude of the tidal force due to the LG, $F_{\rm LG}^{\rm tidal}$, is much smaller than the gravitational pull due to DG, $F_{\rm DG}$. This requires

$$F_{\rm LG}^{\rm tidal} \approx \frac{G\,M_{\rm LG}(R)}{R^2}\,\frac{r}{R} \ll F_{\rm DG} = \frac{G\,M_{\rm DG}(r)}{r^2}, \tag{14}$$

which implies

$$\left(\frac{M_{\rm LG}(R)}{M_{\rm DG}(r)}\right)^{1/3}\frac{r}{R} \ll 1. \tag{15}$$

According to [15], many dwarf galaxies are at least 100 kpc away from the Milky Way and much farther from the Andromeda galaxy (M31). Some of these dwarf galaxies with mass $\sim 10^7 M_\odot$ include Leo T, Leo IV, Canes Venatici I, Canes Venatici II, Hercules, Leo II, Leo V, Leo A, Antlia and DDO 190. Their distances from the Milky Way and M31 are shown in Table 2.