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Non-thermal Dupree diffusivity and shielding effects on atomic collisions in astrophysical turbulent plasmas

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ABSTRACT

The influence of non-thermal Dupree turbulence and the plasma shielding on the electron–ion collision is investigated in astrophysical non-thermal Lorentzian turbulent plasmas. The second-order eikonal analysis and the effective interaction potential including the Lorentzian far-field term are employed to obtain the eikonal scattering phase shift and the eikonal collision cross section as functions of the diffusion coefficient, impact parameter, collision energy, Debye length and spectral index of the astrophysical Lorentzian plasma. It is shown that the non-thermal effect suppresses the eikonal scattering phase shift. However, it enhances the eikonal collision cross section in astrophysical non-thermal turbulent plasmas. The effect of non-thermal turbulence on the eikonal atomic collision cross section is weakened with increasing collision energy. The variation of the atomic cross section due to the non-thermal Dupree turbulence is also discussed.

1. Introduction

The Yukawa-type Debye–Hückel model [1–4] has been widely used for the study of collision and radiation processes in weakly coupled Maxwellian plasmas where the average interaction energy between particles is much less than the average kinetic energy. However, supra-thermal particles are often found in astrophysical plasmas such as solar flares, galactic cosmic ray, planetary magnetospheres, etc. [5–10]. If such supra-thermal particles exist, the collective interaction of plasmas will alter the screening distance and therefore it is required to obtain the effective screening distance by selecting appropriate non-thermal plasma distribution function. We shall employ the generalized Lorentzian distribution function which effectively describes the supra-thermal particles deviated from the Maxwellian distribution. If the plasma is turbulent, the projectile of a moving particle will be affected by the fluctuating electric fields since the response of the field fluctuations plays an important role in the screened binary encounter [11–13]. Then, the effective potential model [14] with the additional Lorentzian far-field term caused by the longitudinal non-linear dielectric function can be applied to describe the potential of a projectile particle in astrophysical turbulent plasmas.

The theoretical work in this paper aims at the explicit investigation of the non-thermal turbulent and the shielding effects on the elastic electron–ion collisions in astrophysical Lorentzian turbulent plasmas. This will provide an insight into the scattering occurrence time in the atomic or radiation processes in astrophysical turbulent plasmas. To this end we employ the second-order eikonal analysis and the turbulence-effective interaction potential to derive the eikonal scattering phase shift and the eikonal collision cross section as functions of the diffusion coefficient, impact parameter, collision energy, Debye length, and spectral index of the Lorentzian plasma. In addition, the non-thermal turbulent shielding effects on the electron–ion collision are compared and discussed with those in astrophysical Maxwellian turbulent plasmas.

2. Effective Shukla–Spatschek potential and second-order eikonal analysis

The generalized Lorentzian (kappa) distribution function in astrophysical plasmas has been represented by the power-law of the velocity in the form [6,7,9,10,15–18]:

$$f_L(v) = \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}\left[\frac{m}{2\pi\kappa E_L(\kappa)}\right]^{3/2}\left[1+\frac{mv^2}{2\kappa E_\kappa(\kappa)}\right]^{-\kappa-1}, \qquad (1)$$

Where v is the particle velocity, κ is the spectral index ($\kappa > 3/2$) of the Lorentzian distribution, $\Gamma(\kappa)$ is the Gamma function with the argument κ, m is the particle mass and $E_\kappa(\kappa)$ is the characteristic energy. The characteristic energy in Lorentzian distribution

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is defined as $E_L(\kappa) = E_T\alpha^2(\kappa)$ where $E_T \equiv k_BT$ with k_B being the Boltzmann constant, T being the plasma temperature and $\alpha(\kappa) \equiv [(\kappa - 3/2)/\kappa]^{1/2}$. The astrophysical Lorentzian distribution function $f_L(v)$ enables us to express quite wide range of velocity distributions from the power-law form to the Maxwellian distribution. When $\kappa \to \infty$ or in the absence of the radiation field the Lorentzian distribution turns out to be the standard Maxwellian distribution such as $f_L(v; \kappa \to \infty) \propto \exp(-mv^2/2E_T)$ [6]. The effective shielding distance [10] $\lambda_L(\kappa)$ in Lorentzian plasmas is represented by $\lambda_L(\kappa) = \lambda_D\beta(\kappa)$, where $\lambda_D(=\sqrt{k_BT/4\pi ne^2})$ is the standard Debye length in Maxwellian plasmas with n being the total density of the plasma, e being the electron charge, and $\beta(\kappa) = [(\kappa - 3/2)/(\kappa - 1/2)]^{1/2}$. The parameter $\beta(\kappa)$ depicts the measure of the fraction of non-thermal population in astrophysical Lorentzian plasmas. In turbulent plasmas, Shukla–Spatschek potential [12] is useful for the effective screened potential of a moving test charge. It can be obtained by employing the longitudinal non-linear plasma dielectric function containing the correction factor [14] $e^{-Dq^2t^3/3}$ owing to the influence of plasma turbulence caused by the fluctuation of electric field, where D is the Dupree diffusion coefficient [19], q is the wave number and t is the time. Then, the effective screened interaction potential $V_{eff}(r, \theta, \kappa)$ between the projectile electron and the ion with charge Ze in astrophysical Lorentzian turbulent plasmas based on the Shukla–Spatschek model [12] with the condition of $v < v_L$ including the non-thermal shielding effect and the far-field term $(r > \lambda_L)$ owing to the influence of electric field fluctuations is represented by

$$V_{eff}(r,\theta,\kappa) = -\frac{Ze^2}{r}\exp\left[-\frac{r}{\lambda_L(\kappa)}\right] - \frac{Ze^2}{r}\frac{2\sqrt{2}}{\sqrt{\pi}}\cos\theta\left[\frac{\lambda_L(\kappa)}{r}\right]^2 \times\left[\frac{v}{v_L(\kappa)}\right]\left[1 - \frac{9}{4}\sqrt{\pi}\frac{D}{v_L^3(\kappa)}r\right], \tag{2}$$

where $r[=(b^2+\eta^2)^{1/2}]$ is the distance between the projectile electron and the target ion, b is the impact parameter, η is the moving distance in the direction of the projectile electron, $v_L(\kappa) \equiv v_T\alpha(\kappa)$, $v_T = \lambda_D\omega_p$, ω_p is the plasma frequency, θ is the angle between $\mathbf{v}$ and $\mathbf{r}$. Eq. (2) is the effective Shukla–Spatschek potential $V_{eff}(r, \theta, \kappa)$ comprised of the Lorentzian effective Debye–Hückel term $[-Ze^2e^{-r/\lambda_L(\kappa)}/r]$ and the additional far-field terms owing to the influence of random fluctuating electric fields in astrophysical Lorentzian turbulent plasmas.

The eikonal scattering phase shift for the electron–ion collision in Lorentzian turbulent plasmas can be analytically calculated by introducing the effective Shukla–Spatschek potential. Let $\varphi(\mathbf{r}) \propto \exp[iS(\mathbf{r})/\hbar]$ be a wave function that satisfies the Hamilton–Jacobi equation [20] for the non-relativistic Schrödinger equation with an interaction potential given by

$$-\frac{i\hbar}{2\mu}\nabla^2S(\mathbf{r}) + \frac{1}{2\mu}[\nabla S(\mathbf{r})]^2 + V(\mathbf{r}) = E, \tag{3}$$

where $S(\mathbf{r})$ is the Hamilton–Jacobi phase function, $\hbar$ is the rationalized Planck constant, μ is the reduced mass, $V(\mathbf{r})$ is the interaction potential and E is the energy of the collision system. The Hamilton–Jacobi phase $S(\mathbf{r})$ in Eq. (3) is obtained as

$$S(\mathbf{r}) \cong \hbar\mathbf{k}_i\cdot\mathbf{r} - \frac{\mu}{\hbar k_i}\int_{-\infty}^{\eta} d\eta' V(\mathbf{r}'), \tag{4}$$

if $|\hbar\nabla^2S(\mathbf{r})| << |[\nabla S(\mathbf{r})]^2|$ and $|V(r_i)|/E < 1$ where η' is the coordinate normal to the momentum transfer $\Delta\mathbf{k}(\equiv \mathbf{k}_f - \mathbf{k}_i)$ with $\mathbf{k}_i$ and $\mathbf{k}_f$ being the incident and the final wave vectors, respectively, and r_i is the interaction range. Then the normalized eikonal wave function [20] $\varphi_{ek}(\mathbf{r})$ is represented by

$$\varphi_{ek}(\mathbf{r}) \cong (2\pi)^{-3/2}\exp\left[i\mathbf{k}_i\cdot\mathbf{r} - i\frac{\mu}{\hbar^2k_i}\int_{-\infty}^{\eta} d\eta' V(\mathbf{r}')\right]. \tag{5}$$

Using the eikonal wave function given in Eq. (5), the eikonal scattering amplitude $f_{ek}(\Delta\mathbf{k})$ would be written by

$$f_{ek}(\Delta\mathbf{k}) = -\frac{\mu}{2\pi\hbar^2}\int d^3\mathbf{r}V(\mathbf{r})\exp\left[i\Delta\mathbf{k}\cdot\mathbf{r} - i\frac{\mu}{\hbar^2k_i}\int_{-\infty}^{\eta'} d\eta' V(\mathbf{r}')\right] \tag{6}$$

which is manipulated to give

$$f_{ek}(\Delta\mathbf{k}) = -ik\int_0^{\infty} dbb\{\exp[i\xi_{ek}(b,k)] - 1\}J_0(\Delta kb) \tag{7}$$

where b is the impact parameter, $\Delta k = 2k\sin(\chi/2)$, χ is the scattering angle between $\mathbf{k}_f$ and $\mathbf{k}_i$, $k \equiv |\mathbf{k}_i| = |\mathbf{k}_f|$ for the elastic collision, $\xi_{ek}(b, k)$ is the total eikonal scattering phase shift, and $J_0(\Delta kb)$ is the zeroth-order first kind Bessel function. If we allow the method of the second-order eikonal analysis [21], one can obtain the total eikonal phase shift as

$$\begin{aligned}\xi_{ek}(b,k) &= \frac{1}{k}\xi_1(b,k) + \frac{1}{k^3}\xi_2(b,k)\\ &= -\frac{\mu}{\hbar^2k}\int_{-\infty}^{\infty} d\eta' V(b,\eta')\\ &\quad + \frac{\mu^2}{2\hbar^4k^3}\int_{-\infty}^{\infty} d\eta'\left[\left(\mathbf{b}\frac{\partial}{b\partial b}\zeta_-(b,\eta') + \boldsymbol{\eta}'\frac{\partial}{\eta'\partial\eta'}\zeta_-(b,\eta')\right)\right.\\ &\quad \left.\cdot\left(\mathbf{b}\frac{\partial}{b\partial b}\zeta_+(b,\eta') + \boldsymbol{\eta}'\frac{\partial}{\eta'\partial\eta'}\zeta_+(b,\eta')\right)\right],\end{aligned} \tag{8}$$

where $\xi_1(b, k)$ and $\xi_2(b, k)$ are the first- and second-order eikonal scattering phases, $\mathbf{b}\partial/b\partial b + \boldsymbol{\eta}'\partial/\eta'\partial\eta'$ is the gradient operator in cylindrical coordinates and the first-term in RHS is the first-order eikonal phase shift, $\zeta_-(b, \eta)$ and $\zeta_+(b, \eta)$ are the functions defined by

$$\zeta_-(b,\eta) = -\frac{\mu}{2\hbar^2}\int_{\eta}^{\infty} d\eta' V(b,\eta'), \tag{9}$$

and

$$\zeta_+(b,\eta) = -\frac{\mu}{2\hbar^2}\int_{-\infty}^{\eta} d\eta' V(b,\eta'), \tag{10}$$

respectively. Plugging the effective Shukla–Spatschek potential $V_{eff}(r, \theta, \kappa)$ into Eq. (8), we finally derive the total eikonal scattering phase shift for the electron–ion collision in astrophysical Lorentzian turbulent plasmas in the form of a dimensionless equation,

$$\begin{aligned}&\xi_{ek}(\bar{b},\kappa,\bar{E},\bar{E}_T,\bar{\lambda}_D,\bar{D})\\ &\cong \frac{2}{\bar{E}^{1/2}}K_0\left(\sqrt{\frac{2\kappa-1}{2\kappa-3}}\frac{\bar{b}}{\bar{\lambda}_D}\right) + \frac{2\kappa(2\kappa-3)}{(2\kappa-1)^2}\frac{\bar{\lambda}_D^4}{2\bar{E}^{1/2}\bar{E}_T}\\ &\quad\times\left[\frac{81\pi}{4}\left(\frac{2\kappa}{2\kappa-3}\right)^3\frac{\bar{D}^2}{\bar{b}^3} - \frac{48}{\sqrt{\pi}}\left(\frac{2\kappa}{2\kappa-3}\right)^{3/2}\frac{\bar{D}}{\bar{b}^4} + \frac{3}{\bar{b}^5}\right],\end{aligned} \tag{11}$$

where $\bar{b}(\equiv b/a_Z)$ is the scaled impact parameter, $a_Z(=a_0/Z)$ is the first Bohr radius of the hydrogenic ion with nuclear charge Ze, $a_0(=\hbar^2/me^2)$ is the Bohr radius of the hydrogen atom, $-e$ is the charge of the electron, $\bar{E}(\equiv \mu v^2/2Z^2Ry)$ is the scaled collision energy, $Ry(=me^4/2\hbar^2 \approx 13.6\,\text{eV})$ is the Rydberg constant, $\bar{E}_T \equiv E_T/2Z^2Ry$, $\bar{D}(\equiv Da_Z/v_T^3)$ is the scaled Dupree diffusion coefficient, and K_0 is the zeroth order modified Bessel function of the second kind. If the influence of non-thermal turbulence is neglected on the collision process, the eikonal scattering phase shift is reduced to $\xi'_{ek}(\bar{b},\kappa,\bar{E},\bar{\lambda}_D) = (2/\bar{E}^{1/2})K_0[\sqrt{(2\kappa-1)/(2\kappa-3)}\bar{b}/\bar{\lambda}_D]$ which contains the non-thermal shielding effect only and is also identical to the case of the first-order eikonal analysis. Hence, the second-order eikonal term is found to cause the non-thermal turbulence effect on