Astroparticle Physics 74 (2016) 73–78

Contents lists available at ScienceDirect

Astroparticle Physics

journal homepage: www.elsevier.com/locate/astropartphys

Absorption cross-section and decay rate of rotating linear dilaton black holes

I. Sakalli*, O.A. Aslan

Physics Department, Eastern Mediterranean University, Famagusta, Northern Cyprus, Mersin 10, Turkey

ARTICLE INFO

Article history:
Received 27 July 2015
Revised 6 October 2015
Accepted 15 October 2015
Available online 22 October 2015

Keywords:
Greybody factor
Absorption cross-section
Decay rate
Dilaton
Axion
Klein–Gordon equation

ABSTRACT

We analytically study the scalar perturbation of non-asymptotically flat (NAF) rotating linear dilaton black holes (RLDBHs) in 4-dimensions. We show that both radial and angular wave equations can be solved in terms of the hypergeometric functions. The exact greybody factor (GF), the absorption cross-section (ACS), and the decay rate (DR) for the massless scalar waves are computed for these black holes (BHs). The results obtained for ACS and DR are discussed through graphs.

1. Introduction

Hawking's semiclassical study [1] on BHs showed that BHs emit particles from their "edge", known as the event horizon. This phenomenon is known as Hawking radiation (HR), named after Hawking. In fact, HR arises from the steady conversion of quantum vacuum fluctuations around into the pairs of particles, one of which escaping at spatial infinity (SI) while the other is trapped inside the event horizon. Calculations of HR reveal a characteristic blackbody spectrum. Thus, putting quantum mechanics and general relativity into the process, BHs become no longer " black" but obey the laws of thermodynamics. However, the spacetime geometry around BH modifies HR by the so-called GFs. Namely, an observer at SI detects not only a perfect black body spectrum but also a modification of this since GFs are dependent upon both geometry and frequency [2].

The first papers of GFs (and its related subjects: ACS and DR) date back to the nineteen-seventies [3–7]. Today, although there exists numerous studies on the subject (see for example [8–10] and references therein), the number of studies regarding rotating BHs is very limited [11–15]. Even there have been very few studies devoted to the NAF rotating BHs [16–18]. This scarcity comes from the technical difficulty of getting exact analytical solution (EAS) to the considered wave equation. In fact, EAS method (see for example [8,19,20]) applies to BH geometries which depend on a radial coordinate.

In this paper, we study GF, ACS, and DR of the RLDBH in 4-dimensions, which is a solution to EMDA theory [21]. To this end, we consider the massless scalar particle and mainly follow the studies of [19,22,23] for using EAS method. It is worth noting that the GF problem (without considering the problem of ACS and DR) of the RLDBH was firstly considered (in broad strokes) in [18]. However, as being stated in the last paragraph of the conclusion of [18], the detailed analysis of GF problem of RLDBH is not completed, and hence it deserves more deeper research. Such an extension is one of the goals of the present paper.

The paper is organized as follows. Section 2 introduces RLDBH geometry, and analyzes the Klein–Gordon equation (KGE) in this geometry. The angular solution of the wave equation is given in Section 3. Section 4 is devoted to the radial solution. GF, ACS, and DR computations are considered in Section 5. The paper ends with a conclusion in Section 6.

2. RLDBH in the EMDA theory and KGE

In the Boyer–Lindquist coordinates, the metric of RLDBH which is the stationary axisymmetric EMDA BH [21] is given by

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + h\left[d\theta^2 + \sin^2\theta\left(d\varphi - a\frac{dt}{h}\right)^2\right], \tag{1}$$

* Corresponding author. Tel.: +90 3926301314.
E-mail addresses: izzet.sakalli@emu.edu.tr, izzet.sakalli@gmail.com (I. Sakalli), onur.aslan@emu.edu.tr (O.A. Aslan).

http://dx.doi.org/10.1016/j.astropartphys.2015.10.005
0927-6505/

with the metric function

$$f = \frac{\Delta}{h}, \tag{2}$$

where $h = rr_0$ in which r_0 is a positive constant. In fact, r_0 is related to the background electric charge and finely tunes the dilaton and axion [21] fields, which are associated with the dark matter [24,25]. Besides

$$\Delta = (r - r_+)(r - r_-), \tag{3}$$

where r_+ and r_- are the outer (event) and inner (Cauchy) horizons, respectively, given by the zeros of g_{tt}:

$$r_\pm = M \pm \sqrt{M^2 - a^2}, \tag{4}$$

where M is associated with the quasilocal mass (M_{QL}) via $M = 2M_{QL}$ [26], and a denotes the rotation parameter, which also tunes the both dilaton and axion fields [21]. One can immediately see from Eq. (4) that having a BH is conditional on $M \geq a$. Otherwise, there is no horizon and the spacetime corresponds to a BH with a naked singularity at $r = 0$. The angular momentum (J) and a are related in the following way: $ar_0 = 2J$. When a vanishes, RLDBH reduces to its static form, the so-called linear dilaton black hole (LDBH) metric. For studies of LDBH, the reader is referred to [27–38]. The area (A_{BH}), Hawking temperature (T^H_{RLDBH}) and angular velocity (Ω_H) at the horizon are found to be [23]

$$A_{BH} = 4\pi r_0 r_+, \tag{5}$$

$$T^H_{RLDBH} = \frac{\kappa}{2\pi} = \left.\frac{\partial_r f}{4\pi}\right|_{r=r_+} = \frac{r_+ - r_-}{4\pi r_0 r_+}, \tag{6}$$

$$\Omega_H = 2\frac{J}{r_0^2 r_+} = \frac{a}{r_0 r_+}. \tag{7}$$

It is worth noting that T^H_{RLDBH} vanishes at the extremal limit $M = a$, i.e., $r_+ = r_-$. Moreover, as $a \to 0$ ($r_- \to 0$), $T^H_{RLDBH} \to T^H_{LDBH} = \frac{1}{4\pi r_0}$ which is independent of the mass of the BH, and points an isothermal HR [27,32].

The massless KGE equation in a curved spacetime is given by

$$\partial_\mu\left(\sqrt{-g}g^{\mu\upsilon}\partial_\upsilon\Psi\right) = 0. \tag{8}$$

It is straightforward to show that Eq. (8) separates for the solution ansatz of the form $\Psi = \psi(r,\theta)e^{i(m\varphi-\omega t)}$, where m and ω are constant associated with rotation in the φ-direction and frequency, respectively. Thus, we can obtain the following master equation:

$$\partial_r(\Delta\partial_r\psi) + \frac{\partial_\theta(\sin\theta\partial_\theta\psi)}{\sin\theta} + \left[\frac{(h\omega - am)^2}{hf} - \left(\frac{m}{\sin\theta}\right)^2\right]\psi = 0. \tag{9}$$

If we let $\psi = R(r)\Theta(\theta)$, Eq. (9) is separated into radial and angular equations as follows

$$\Delta\partial_r(\Delta\partial_r R) + \left[(h\omega - am)^2 - \lambda\Delta\right]R = 0, \tag{10}$$

$$\partial_\theta(\sin\theta\partial_\theta\Theta) + \sin\theta\left[\lambda - \left(\frac{m}{\sin\theta}\right)^2\right]\Theta = 0, \tag{11}$$

where λ denotes the eigenvalue.

3. Solution of the angular equation

In order to have the general solution to Eq. (10), we introduce a new dimensionless variable as follows:

$$z = \frac{1 - \cos\theta}{2}, \tag{12}$$

so that Eq. (11) becomes

$$z(1-z)\partial_{yy}\Theta + (1-2z)\partial_y\Theta + \left[\frac{4z(z-1)\lambda + m^2}{4z(z-1)}\right]\Theta = 0. \tag{13}$$

One can rewrite the factor of Θ in the third term of Eq. (13) as follows:

$$\frac{4\lambda z(z-1) + m^2}{4z(z-1)} = \lambda - \frac{m^2}{4z} + \frac{m^2}{4(z-1)}, \tag{14}$$

Letting

$$\Theta = \left(\frac{1-z}{z}\right)^{\frac{m}{2}}\Phi(z), \tag{15}$$

Eq. (13) is transformed into

$$z(1-z)\partial_{zz}\Phi + \left[\bar{c} - (1 + \bar{a} + \bar{b})z\right]\partial_z\Phi - \bar{a}\bar{b}\Phi = 0. \tag{16}$$

The above equation resembles the standard hypergeometric equation [39] whose solution is given by

$$\Phi = C_1 F\left(\bar{a}, \bar{b}; \bar{c}; z\right) + C_2 z^{1-\bar{c}} F\left(\bar{a} - \bar{c} + 1, \bar{b} - \bar{c} + 1; 2 - \bar{c}; z\right), \tag{17}$$

where $F\left(\bar{a}, \bar{b}; \bar{c}; z\right)$ is the standard (Gaussian) hypergeometric function [39], and C_1, C_2 are integration constants. By performing a few algebraic manipulations, one obtains the following identities

$$\bar{a} = \frac{1}{2}(1 - \sqrt{4\lambda + 1}), \tag{18}$$

$$\bar{b} = 1 - \bar{a} = \frac{1}{2}(1 + \sqrt{4\lambda + 1}), \tag{19}$$

$$\bar{c} = 1 - m. \tag{20}$$

Consequently, the general angular solution reads

$$\Theta = C_1\left(\frac{1-z}{z}\right)^{\frac{m}{2}} F\left(\bar{a}, \bar{b}; \bar{c}; z\right) + C_2\left[z(1-z)\right]^{\frac{m}{2}} \times F\left(\bar{a} - \bar{c} + 1, \bar{b} - \bar{c} + 1; 2 - \bar{c}; z\right). \tag{21}$$

However, we need the normalized angular solution [40]. For this purpose, we initially set $C_2 = 0$, and assign the eigenvalue to

$$\lambda = l(l+1), \tag{22}$$

where $l = 0, 1, 2, 3 \ldots$. Using the following transformation [41]:

$$F(\bar{a}, 1 - \bar{a}; \bar{c}; z) = \left(\frac{-z}{1-z}\right)^{(1-\bar{c})/2} P(-\bar{a}, 1 - \bar{c}, 1 - 2z), \tag{23}$$

where P denotes the associated Legendre polynomials [39], we re-express

$$\Theta = \widehat{C}_1 P(l, m, 1 - 2z), \tag{24}$$

which can rewritten as

$$\Theta = \widehat{C}_1 P(l, m, \cos\theta), \tag{25}$$

where $\widehat{C}_1 = C_1(-1)^{\frac{m}{2}}$. Employing the orthonormality relation [40] for the associated Legendre functions and taking $e^{im\varphi}$ into account, we obtain the physical angular solution in terms of the spherical harmonics [41]:

$$Y_{l,m}(\theta,\varphi) = e^{im\varphi}\sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}}P(l, m, \cos\theta), \tag{26}$$

where the index l corresponds to the well-known azimuthal quantum number, and m denotes the magnetic quantum number (integer) with $-l \leq m \leq l$.