Astroparticle Physics 59 (2014) 4–11

Contents lists available at ScienceDirect

Astroparticle Physics

journal homepage: www.elsevier.com/locate/astropart

An extended Heitler–Matthews model for the full hadronic cascade in cosmic air showers

J.M.C. Montanus *

Nikhef, Science Park 105, 1098 XG Amsterdam, The Netherlands

ARTICLE INFO

Article history:
Received 30 December 2013
Received in revised form 24 March 2014
Accepted 27 March 2014
Available online 5 April 2014

Keywords:
Cosmic rays
Air showers
Heitler–Matthews model
Hadronic cascade

ABSTRACT

The Heitler–Matthews model for hadronic air showers will be extended to all the generations of electromagnetic subshowers in the hadronic cascade. The analysis is outlined in detail for showers initiated by primary protons. For showers initiated by iron primaries the part of the analysis is given for as far as it differs from the analysis for a primary proton. Predictions for shower sizes and the depth of maximum shower size are compared with results of Monte Carlo simulations. The depth of maximum as it follows from the extrapolation of the Heitler–Matthews model restricted to the first generation of electromagnetic subshowers is too small with respect to Monte Carlo predictions. It is shown that the inclusion of all the generations of electromagnetic subshowers leads to smaller predictions for the depth of maximum and to smaller predictions for the elongation rate. The discrepancy between discrete model predictions and Monte Carlo predictions for the depth of maximum can therefore not be explained from the number of generations that is taken into consideration. An alternative explanation will be proposed.

1. Introduction

A simplified description of the longitudinal evolution of electromagnetic showers is given by the Heitler model [1]. Starting with a primary particle of energy E_0, the number of particles doubles every splitting length $d = \lambda_r \ln 2$, where the radiation length λ_r is about 37 g cm^{-2}. The doubling stops when the energy per particle is equal to the critical energy $\xi_c^e \approx 85$ MeV. The resulting Heitler profile is

$$N(X) = \begin{cases} 2^{X/d}, & X \leqslant n_c^e d; \\ 0, & X > n_c^e d, \end{cases} \tag{1}$$

where n_c^e is maximum the number of steps: $n_c^e \ln 2 = \ln(E_0/\xi_c^e)$.

A Heitler model for the hadronic cascade in air showers has been constructed by Matthews [2]. The Heitler–Matthews model is useful for the explanation of hadronic cascades as well as for the analytical derivation of relations between quantities as primary energy, muon number, electron number and depth of maximum shower size [3–5]. For the prediction of the number of charged particles it is assumed that each hadronic interaction results in $M_{\rm ch} = 10$ charged pions and $\frac{1}{2}M_{\rm ch} = 5$ neutral pions. That is, the total multiplicity M is equal to 15. The neutral pions initiate electromagnetic subshowers when they decay into photons. For the prediction for the depth of maximum shower size, restricted to the first generation of electromagnetic subshowers, the multiplicity and interaction length are parameterized by the energy of the interaction.

The atmosphere is divided into layers of atmospheric thickness $d_{\rm I}$. After the traversing of each layer the number of charged pions is assumed to be $M_{\rm ch}$ times larger if $d_{\rm I} = \lambda_{\rm I} \ln 2$, where $\lambda_{\rm I} = 120$ g cm^{-2} is the interaction length of strongly interacting pions. Consequently, after n layers the number of charged pions is $(M_{\rm ch})^n$. The energy per pion is

$$E_{\pi,n} = \frac{E_0}{M^n}. \tag{2}$$

The stopping energy is estimated on the basis of the finite lifetime of the pions in the atmosphere. For this it suffices to consider the approximate relation between atmospheric depth and height:

$$X(h) = 1030 \cdot e^{-h/8} \quad \leftrightarrow \quad h(X) = 8 \ln(1030/X), \tag{3}$$

where X is the depth in g cm^{-2} and h is the height in km. Neutral pions decay almost immediately into two photons, $c\tau = 25$ nm [6]. Each resulting photon starts an electromagnetic shower. The decay length of the charged pions is $\gamma c\tau$, where $c\tau = 7.8$ m [6]. The decay length is of the order of a kilometer because of the relativistic time dilation. As a consequence charged pions may interact with the atmosphere and propagate the hadronic shower, before

* Tel.: +31 (0)20 592 5126.
E-mail addresses: hans.montanus@wxs.nl, hansm@nikhef.nl

http://dx.doi.org/10.1016/j.astropartphys.2014.03.010
0927-6505/

decay. If the probability for decay in the next layer is larger than the probability of a hadronic interaction, the pions are assumed to decay and the cascade stops. This happens after n_c layers. The corresponding energy of the decaying charged pions, the stopping energy ξ_c^π, follows from

$$\xi_c^\pi = \frac{E_0}{M^{n_c}}. \tag{4}$$

The stopping energy turns out to be around 20 GeV.

2. Model parameters

In this paper the Heitler–Matthews model is extended to all the generations of pions in the shower. The complete analysis will be improved by consequently taking the multiplicity M and interaction length λ_I to depend on the energy of the hadron in the shower. One of the consequences is that the thickness of the cascade layers increases with depth, see Fig. 1.

We will take the energy dependence of the π–air multiplicity and interaction length to be given by Monte Carlo event generators based on QCD and parton models. The calculated π–air charged multiplicity, see Fig. 5 of [7], Fig. 7 of [8] and Fig. 5 of [9], suggests the relation

$$M_{ch} \approx 0.1 \cdot E^{0.18}, \tag{5}$$

where E is the energy in eV. Taking the ratio of the charged and neutral pions as 2 : 1, we have for the total multiplicity

$$M \approx 0.15 \cdot E^{0.18}. \tag{6}$$

It should be emphasized that the relation between multiplicity and energy is rather uncertain since different models predict different multiplicities. In particular for large energies the differences can be large, even more than 100%. From Fig. 5 of [7] and Fig. 7 of [8] we see that the pion multiplicity does not differ substantially from the proton multiplicity. We therefore will use the relations (5) and (6) for both the multiplicity in proton–air (p–air) and pion–air (π–air) interactions. A parameterization with other values for the constants will, of course, affect the results quantitatively. It does, however, not alter the results qualitatively.

Both the p–air and π–air inelastic cross sections grow with energy. The p–air inelastic cross section at large energies obtained from observations of extensive air showers are in good agreement with QGSJET predictions [10–13]. For the present analysis we will therefore use the QGSJET predictions for the p–air inelastic cross section. We will also use the QGSJET predictions for the π–air inelastic cross section [8,14]. From these cross sections approximations for the energy dependent interaction lengths can be derived which are sufficiently accurate for our purpose. For π–air this is:

$$\lambda_{\pi\text{–air}}\ [\mathrm{g\,cm^{-2}}] \approx 200 - 3.3\ln(E[\mathrm{eV}]). \tag{7}$$

For p–air this is

$$\lambda_{\text{p–air}}\ [\mathrm{g\,cm^{-2}}] \approx 145 - 2.3\ln(E[\mathrm{eV}]). \tag{8}$$

For the hadronic cascade Matthews and Hörandel take $d_I = \lambda_I \ln 2$ as the relation between layer thickness and interaction length [2,3]. This might have been motivated by the expression $\lambda_r \ln 2$ for the splitting length in the electromagnetic cascade. There the ratio ln 2 results from the translation of the radiation length to the splitting length. In an intermediate model for electromagnetic showers the splitting length, $\lambda_r \ln 2$, is effectively used as the electromagnetic interaction length [15]. For the hadronic cascade, however, there is no reason for the ratio ln 2 since λ_I is already an interaction length. As a consequence the thickness of the interaction layer in hadronic cascades is equal to the interaction length. For hadronic showers we will therefore use the relation $d_I = \lambda_I$.

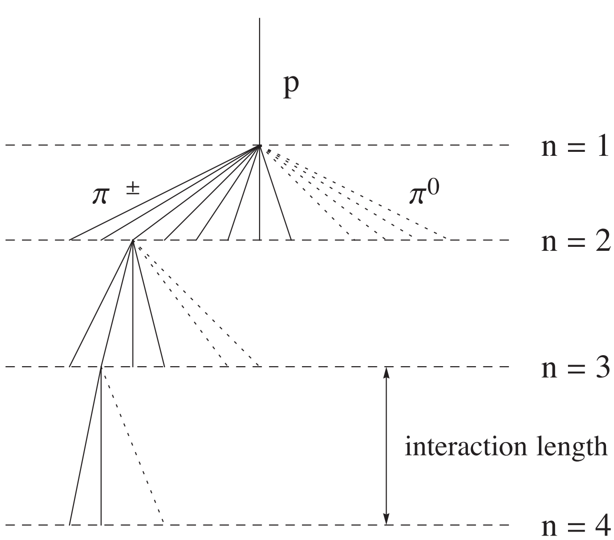

Fig. 1. The hadronic cascade for energy dependent interaction lengths.

3. The hadronic cascade for a primary proton

Now we consider a hadronic cascade where the hadronic particles interact after having traversed a layer of atmosphere. The thickness of each layer will be taken equal to the actual interaction length as given by (7) or (8). After each interaction M pions are produced as given by (6). In accordance with the Heitler model for electromagnetic showers, the energy is assumed to be equally divided over the particles produced. After each interaction the new energy of the charged hadrons then follows from a successive application of the equation

$$E_{j+1} = \frac{E_j}{M(E_j)}. \tag{9}$$

Starting with a primary proton with energy E_0 the energy of the particles after the first interaction is

$$E_1 = \frac{E_0}{0.15 \cdot E_0^{0.18}} \approx 6.7 \cdot E_0^{0.82}. \tag{10}$$

After the second interaction this is

$$E_2 = \frac{E_1}{0.15 \cdot E_1^{0.18}} \approx 6.7 \cdot E_1^{0.82} \approx 6.7^{1.82} \cdot E_0^{0.82^2}. \tag{11}$$

Repeating the iteration we find for the energy per particle after n interactions

$$E_n = 6.7^{\alpha_n} \cdot E_0^{\beta_n}, \tag{12}$$

where

$$\alpha_n = \frac{1 - 0.82^n}{1 - 0.82}, \quad \beta_n = 0.82^n. \tag{13}$$

For the interaction lengths we obtain for the primary proton, $n = 0$,

$$\lambda_0 = \lambda_{\text{p–air}}(E_0) = 145 - 2.3\ln(E_0) \tag{14}$$

and for the produced pions, $n \geqslant 1$,

$$\lambda_n = \lambda_{\pi\text{–air}}(E_n) = 200 - 3.3\ln(E_n). \tag{15}$$

With the substitution of (12) this is

$$\lambda_n = 200 - 6.3\alpha_n - 3.3\beta_n \ln(E_0), \quad n \geqslant 1. \tag{16}$$