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On the stability of a class of radiating viscous self-gravitating stars with axial symmetry

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ABSTRACT

This study investigates the stability of a class of radiating viscous self-gravitating stars with axial symmetry having anisotropic pressure. We use perturbation technique to establish the perturbed form of the Einstein field equations and dynamical equations. The instability range in the Newtonian and post-Newtonian eras has been analyzed by constructing the collapse equation. It is found that the adiabatic index has a key role in the discussion of instability ranges which depends upon the physical parameters, i.e., energy density, anisotropic pressure and shear viscosity of the fluid and heat flux. We conclude that the shear viscosity decreases the instability range and makes the system more stable.

1. Introduction

Gravitational theories have some outstanding issues in the discussion of instabilities of massive stars and have gone through extensive developments in recent years. The formation of self-gravitating objects and their evolution is associated with dynamical instability which is usually generalized with anisotropic matter distribution. Pressure anisotropy is identified as one of the main factors in stellar models which makes the fluid imperfect through different mechanisms. Herrera and Santos [1] investigated pressure anisotropy for self-gravitating objects whose role has been examined in static stellar configurations by different authors [2]. Chan et al. [3] analyzed the dynamical instability of anisotropic matter in the collapse scenario. Many phenomena such as the solid core, phase transition, mixture of two fluids, slow rotation and pion condensation can generate anisotropy in the star model [1,2,4]. The anisotropy of the system increases with the inclusion of shear viscosity in the matter distribution.

Many investigations are devoted to study the effects of viscosity in the matter distribution which has physical justification [5]. A normal star is in hydrostatic equilibrium due to the balance of external and internal forces. When one of these forces overcomes the other than stability of the star is affected and collapse may take place. Chan [6] studied the effects of viscosity on the stability of collapsing fluid distribution. Joshi et al. [7] found that the shearing effects slow down the apparent horizon formation and hence the collapse rate due to irregular final stages of the collapse.

During the formation and evolution of astrophysical objects, energy radiates in large amount at different states in the form of photons and neutrinos. This radiated energy increases gradually and is described by two approximations: one is the diffusion approximation and the other is free streaming approximation. We have discussed the evolution of shear and expansion through Raychaudhuri equation for radiating viscous fluid [8]. Sharma and Tikekar [9] analyzed the collapse of radiating spherical star in the form of heat flux with the induction of inhomogeneous perturbations and anisotropic pressure in FRW geometry. Recently, we have investigated some exact solutions of non-viscous heat conducting collapsing fluid and examined surface temperature of the system at large past time [10].

In general relativity, the study of perturbation has gained interest due to two major reasons out of which one corresponds to the stability of self-gravitating objects. The important aspect is the instability of linear perturbations in higher order theories [11] but one cannot find that what extent the linear perturbation can decide the stability issue. Chandrasekhar [12] is known due to his pioneer work on spherical star with perfect fluid to understand the dynamical instability. The leading term which describes the instability range is the Γ (adiabatic index) factor for Newtonian (N) regime. Sorkin et al. [13] explored the instability of radiating spherical star. Herrera et al. [14] extended Chandrasekhar work for dissipative fluids. Boehmer and Harko [15] examined the instability of spherical system with cosmological constant in the presence of perfect fluid. Bisnovatyi-Kogan and Tspuko [16] observed that the loss of stability would lead to collapse yielding a black hole or neutron star.

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Herrera et al. [17] examined the instability of spherical system with anisotropic pressure and expansion-free case through linear perturbation. Sharif and Bhatti [18] investigated shearfree collapse of charged spherical star and found that the shearfree condition makes the system more stable by slowing down the collapse rate. Sharif and Yousaf [19,20] discussed the instability ranges in $f(R)$ gravity using perturbation scheme of collapsing models and found that Γ is a key factor in the dynamical instability of self-gravitating fluids. The deviations from spherical systems have great significance in the instability analysis. In recent papers, we have explored the effects of electromagnetic field on the instability ranges of anisotropic expansion-free [21] and radiating [22] cylinders. It is found that Γ does not play any role in the expansion-free case but for radiating cylinder, the instability range depends upon Γ.

It is known that a rotating star is more stable against collapse than the non-rotating and stationary rotating objects carry axial symmetry. Stars in nature are usually rotating based on rotational instabilities in non-axis symmetry. The static axially symmetric solutions with and without the effects of electromagnetic field are formulated in literature which corresponds to the Schwarzschild metric and charged solution, respectively in spherical limit [23]. Recently, we have analyzed the role of Γ in the instability range of a restricted class of non-static axial symmetry with anisotropic pressure [24].

This study extends the above work to radiating anisotropic viscous matter distribution for the instability regions of a restricted class of non-static axially symmetric spacetime. The paper is organized as follows. In the next section, we describe the fluid configuration compatible with the axial symmetry, the corresponding field equations as well as the dynamical equations. Section 3 explores the dynamics of axial symmetry through perturbation technique and formulates the collapse equation. We study instability regions in N and post-Newtonian (pN) eras in Section 4. Finally, we conclude our results in Section 5.

2. Matter distribution and field equations

We consider a restrictive class of non-static axial symmetry which excludes explicitly $dtd\phi$ terms representing the rotations around the symmetry axis as well as the reflection terms. Inclusion of "reflection" and "rotation" terms with four independent metric functions would make our analysis quiet complicated which is very difficult to handle analytically. For the sake of convenience, we exclude these terms and the corresponding non-static axially symmetric spacetime in spherical coordinates reduces to [24]

$$ds^2 = -A^2(t,r,\theta)dt^2 + B^2(t,r,\theta)(dr^2 + r^2d\theta^2) + C^2(t,r,\theta)d\phi^2. \quad (1)$$

We assume that the system is filled with radiating anisotropic matter suffering with shear viscosity in its flow. The energy–momentum tensor for such a system is defined as

$$T^{(m)}_{\alpha\beta} = (\mu + P)V_\alpha V_\beta + Pg_{\alpha\beta} - 2\eta\sigma_{\alpha\beta} + q_\alpha V_\beta + q_\beta V_\alpha + \Pi_{\alpha\beta}, \quad (2)$$

where

$$\Pi_{\alpha\beta} = (P_{yy} - P_{zz})\left(L_\alpha L_\beta - \frac{1}{3}h_{\alpha\beta}\right) + (P_{xx} - P_{zz}) \times \left(K_\alpha K_\beta - \frac{1}{3}h_{\alpha\beta}\right) + 2P_{xy}K_{(\alpha}L_{\beta)},$$

$$P = \frac{1}{3}(P_{xx} + P_{yy} + P_{zz}), \quad h_{\alpha\beta} = g_{\alpha\beta} + V_\alpha V_\beta,$$

P_{xx}, P_{yy}, P_{zz}, μ are different pressures and energy density, respectively, with $P_{xy} = P_{yx}$ and $P_{xx} \neq P_{yy} \neq P_{zz}$. Also, V_α, L_α, K_α, q_α, η, $\sigma_{\alpha\beta}$ are the four velocity, unit four-vectors, heat flux, coefficient of shear viscosity and the shear tensor, respectively, α, β are the Lorentz indices. In comoving coordinate system, these quantities have the following form

$$V_\alpha = -A\delta^0_\alpha, \quad K_\alpha = B\delta^1_\alpha, \quad L_\alpha = Br\delta^2_\alpha, \quad q^\alpha = qB^{-1}\delta^\alpha_1. \quad (3)$$

The non-vanishing components of the kinematical variables [25], i.e., the expansion scalar Θ, the four acceleration a_α and the shear tensor $\sigma_{\alpha\beta}$, related with the given fluid distribution turn out to be

$$\Theta = \left(\frac{\dot{C}}{C} + \frac{2\dot{B}}{B}\right)\frac{1}{A}, \quad a_1 = \frac{A'}{A}, \quad a_2 = \frac{A_\theta}{A},$$

$$\sigma_{11} = \frac{1}{3}B^2\sigma, \quad \sigma_{22} = \frac{1}{3}r^2B^2\sigma, \quad \sigma_{33} = -\frac{2}{3}C^2\sigma,$$

$$\sigma = -\frac{1}{A}\left(\frac{\dot{C}}{C} - \frac{\dot{B}}{B}\right),$$

where dot, prime and subscript θ indicate differentiation with respect to t, r and θ, respectively. The Misner–Sharp mass function [26] corresponding to Eq. (1) yields

$$m = \frac{r^3B}{2}\left(\frac{\dot{B}^2}{A^2} - \frac{B_\theta^2}{r^2B^2} - \frac{2B'}{rB} - \frac{B'^2}{B^2}\right).$$

The scalar curvature corresponding to (1) is

$$R = 2\left[\frac{1}{B^2}\left\{\frac{A''}{A} + \frac{A'C'}{AC} + \frac{B''}{B} + \frac{C''}{C} + \frac{1}{r}\left(\frac{C'}{C} + \frac{B'}{B} - \frac{A'}{A}\right) - \frac{B'^2}{B^2} + \frac{1}{r^2}\left(\frac{A_{\theta\theta}}{A} + \frac{B_{\theta\theta}}{B} - \frac{B_\theta^2}{B^2} + \frac{A_\theta C_\theta}{AC} + \frac{C_{\theta\theta}}{C}\right)\right\} + \frac{1}{A^2}\left(2\frac{\dot{A}\dot{B}}{AB} - 2\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{B}^2}{B^2} - 2\frac{\dot{B}\dot{C}}{BC}\right)\right]. \quad (4)$$

The Einstein field equations provide the following set of equations

$$\kappa\mu = \frac{1}{A^2}\left(\frac{\dot{B}^2}{B^2} + 2\frac{\dot{B}\dot{C}}{BC}\right) - \frac{1}{B^2}\left[\frac{B''}{B} + \frac{1}{r}\left(\frac{B'}{B} + \frac{C'}{C}\right) - \left(\frac{B'}{B}\right)^2 + \frac{C''}{C} - \frac{1}{r^2}\left\{\left(\frac{B_\theta}{B}\right)^2 - \frac{B_{\theta\theta}}{B} - \frac{C_{\theta\theta}}{C}\right\}\right], \quad (5)$$

$$-\kappa qAB = \frac{A'\dot{B}}{AB} - \frac{\dot{C}'}{C} + \frac{\dot{B}B'}{B^2} + \frac{\dot{C}A'}{CA} - \frac{\dot{B}'}{B} + \frac{\dot{B}C'}{BC}, \quad (6)$$

$$0 = \frac{A_\theta\dot{C}}{AC} - \frac{\dot{B}_\theta}{B} + \frac{B_\theta\dot{B}}{B^2} + \frac{A_\theta\dot{B}}{AB} + \frac{\dot{B}C_\theta}{BC} - \frac{\dot{C}_\theta}{C}, \quad (7)$$

$$\kappa\left(P_{xx} - \frac{2}{3}\eta\sigma\right) = \frac{1}{A^2}\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} - \frac{\ddot{B}}{B} - \frac{\dot{B}\dot{C}}{BC} - \frac{\ddot{C}}{C}\right) + \frac{1}{B^2}\left[\frac{A'B'}{AB} + \frac{A'C'}{AC} + \frac{B'C'}{BC} + \frac{1}{r}\left(\frac{C'}{C} + \frac{A'}{A}\right) + \frac{1}{r^2}\left(\frac{A_{\theta\theta}}{A} + \frac{C_{\theta\theta}}{C} - \frac{A_\theta B_\theta}{AB} - \frac{B_\theta C_\theta}{BC} + \frac{A_\theta C_\theta}{AC}\right)\right], \quad (8)$$

$$\kappa\left(P_{yy} - \frac{2}{3}\eta\sigma\right) = \frac{1}{A^2}\left(\frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}\dot{B}}{AB} - \frac{\ddot{C}}{C} - \frac{\ddot{B}}{B} - \frac{\dot{B}\dot{C}}{BC}\right) + \frac{1}{B^2}\left[\frac{C''}{C} + \frac{A''}{A} + \frac{A'C'}{AC} - \frac{B'C'}{BC} - \frac{A'B'}{AB} + \frac{1}{r^2}\left(\frac{A_\theta B_\theta}{AB} + \frac{A_\theta C_\theta}{AC} + \frac{B_\theta C_\theta}{BC}\right)\right], \quad (9)$$