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Testing energy non-additivity in white dwarfs

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ABSTRACT

We consider a particular effect which can be expected in scenarios of deviations from special relativity induced by Planckian physics: the loss of additivity in the total energy of a system of particles. We argue about the necessity to introduce a length scale to control the effects of non-additivity for macroscopic objects and consider white dwarfs as an appropriate laboratory to test this kind of new physics. We study the sensitivity of the mass-radius relation of the Chandrasekhar model to these corrections by comparing the output of a simple phenomenological model to observational data of white dwarfs.

1. Introduction

The relativistic energy–momentum dispersion relation of particles, $E^2 - \vec{p}^2 = m^2$, is involved in every particle physics measurement, and, as so, it has been tested in countless occasions. This has been done with large precision however only in the relatively "low-energy" regime we are familiar with (energies at or below the Fermi scale, of $\mathcal{O} \sim 100$ GeV). In recent times, astrophysics has also offered the opportunity to test this relation at much higher energies [1]. In fact, the loss of Lorentz invariance is a rather generic feature of quantum gravity developments [2], so that modifications to this dispersion relation are expected at energies close to the Planck mass M_P. However, the presence of amplifying mechanisms, such as threshold effects [3] or galactic distances of propagation [4], as well as high-precision experiments [5] makes possible to consider a quantum gravity phenomenology at energies much lower than M_P [6].

In order to test this relation, it is quite common to assume corrections of order $1/\Lambda$ as the dominant effect if the energies involved are much smaller than the ultraviolet scale Λ which controls the new physics beyond special relativity (SR). In a quantum gravity context, one would take $\Lambda = M_P$, but the idea of a modified dispersion relation (MDR) is more general than that. A typical parametrization in which rotational invariance is maintained and the corrections can be expanded in powers of the momenta and the inverse of Λ is

$$E^2 - \vec{p}^2 + \frac{\alpha_1}{\Lambda}E^3 + \frac{\alpha_2}{\Lambda}E\vec{p}^2 = m^2, \tag{1}$$

where α_1 and α_2 are two coefficients of order 1 (that is, Λ signals the energy scale at which the corrections are of order 1).

Of course, energy is not an observer-invariant quantity, so that when one is comparing the typical "energy" scale of an experiment with the scale Λ, one is either assuming a particular (laboratory) frame, or considering an invariant quantity (such as the invariant mass or the energy in the center of mass system in special relativity). In the case of a *violation* of Lorentz invariance, the only possibility is to consider a particular, privileged, system of reference [7]. However, it is also possible to go beyond SR by considering a *deformation* of Lorentz invariance. This is the case of doubly special relativity (DSR) models [8]. The phenomenology of Lorentz invariance violation (LIV) and DSR models are in general quite different, since the existence of a relativistic principle in the latter forces to add to a MDR also a modification in the composition laws (the sum of the energies and momenta) of a multiparticle system. A general parametrization of modified composition laws (MCL) at order $1/\Lambda$ is

$$E_1 \oplus E_2 = E_1 + E_2 + \frac{\beta_1}{\Lambda}E_1E_2 + \frac{\beta_2}{\Lambda}\vec{p}_1 \cdot \vec{p}_2 \tag{2}$$

$$\vec{p}_1 \oplus \vec{p}_2 = \vec{p}_1 + \vec{p}_2 + \frac{\gamma_1}{\Lambda}E_1\vec{p}_2 + \frac{\gamma_2}{\Lambda}\vec{p}_1E_2 + \frac{\gamma_3}{\Lambda}\vec{p}_1 \times \vec{p}_2. \tag{3}$$

again with coefficients (β_i, γ_i) of order 1.

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The idea of MCL can also be considered in more general contexts than those of DSR models. It can be shown [9] that MCL, that is, nonlinear corrections to additive energy–momentum composition laws, cause the locality property of interactions between particles to be lost for a general observer. This in fact agrees with the notion of "relative locality" that has recently been proposed in terms of a geometric interpretation of the departures from SR kinematics [10]. In particular, both a MDR and MCL may be present as signals of a LIV. What the existence of a relativity principle makes is to establish specific relations between the coefficients appearing in the MDR and the MCL. In fact, as it is shown in Ref. [11] the β and γ coefficients of the MCL determine the α coefficients in the MDR in a relativistic theory beyond SR:

$$\alpha_1 = -\beta_1 \quad \alpha_2 = \gamma_1 + \gamma_2 - \beta_2. \tag{4}$$

Since the idea of MCL is very general, compulsory in DSR scenarios and possible in LIV scenarios, we want to address in this paper the physical implications of an energy non-additivity. Naively one would think that the best place to look for signals of an energy non-additivity would be systems of a large number of particles; however, for a sufficiently large system, the nonlinear terms cease to be small corrections contradicting, for example, the well-known physics of macroscopic objects. This difficulty is related with the general question of how the correction to SR in the microscopic domain affects macroscopic physics, which in the DSR context, where MCL are a necessary ingredient of the models, it is known as the "soccer-ball problem". This problem by itself deserves a general discussion of the different alternatives to solve it and a comparison of the required assumptions and their plausibility [12]. We will just note here that if gravity or the structure of space–time is in the origin of this non-additivity, then it is natural to think that the key for these corrections to be large should be not a large number of particles, but a large number of particles which are close enough. That is, the relevant variable controlling the size of the non-additivity should be the density of the system. Incorporating this idea would immediately provide a possible solution to the soccer-ball problem, since the effects would be important not just for macroscopic objects, but for objects of high enough density, as it was already remarked in a different context but with similar implications by Ref. [13]. The requirement of a high energy density is in contrast with other proposals to observe effects of the quantization of space–time in macroscopic objects [14], which have been questioned by the results of Ref. [15].

In Section 2 of the paper we will make the previous idea explicit, by introducing the notion of a "coherence length" for the non-additivity corrections. Then we will explore the possible systems where these corrections could be tested, and conclude that white dwarfs are a natural candidate. In fact modifications in white dwarf physics owing to departures of Lorentz invariance have been considered previously [16–18]. We will review in Section 3 the main conclusions of these works and will remark their differences with the ideas here explored. In Section 4 we will present a phenomenological model including non-additivity corrections for the energy of a white dwarf and will calculate how the mass-radius relation that can be extracted from the Chandrasekhar model gets modified. A comparison with experimental data to obtain bounds on the parameters of the model (essentially, the coherence length) will be done in Section 5, showing the sensitivity of white dwarfs to this new physics. Finally, we will present our conclusions in Section 6.

2. Energy non-additivity: coherence length

Our aim is to explore possible observational consequences of new physics parameterized at a kinematical level by a modification (non-additivity) of the energy composition law. As explained in the Introduction, this non-additivity emerges naturally in specific scenarios related to quantum gravity effects, but we will just consider it as a phenomenological model for the new physics, without entering into the physical mechanisms at work. A departure from locality [9] or a hidden effective interaction as a remnant of a quantum structure of space–time are examples of possible ideas related to the origin of an energy non-additivity.

Let us assume that the total energy of a two particle system is given by Eq. (2), which contains nonlinear corrections to the simple addition of energies of order $1/\Lambda$. Λ is a high (ultraviolet) energy scale which may be as large as the Planck energy scale if the new physics we are considering is a remnant of quantum gravity. Since we will never have direct access to particles with energies of the order of the Planck energy scale, in order to have observable consequences of the non-additivity we have to rely on the amplification of the kinematic modification that happens in a many particle system, for which Eq. (2) is generalized to (see Ref. [11]):

$$\sum_{\oplus} E_i \equiv \sum_i E_i + \sum_{i<j} \left(\frac{\beta_1}{\Lambda} E_i E_j + \frac{\beta_2}{\Lambda} \vec{p}_i \cdot \vec{p}_j \right). \tag{5}$$

Let us think, for the sake of simplicity, of an object made of N particles with the same energy E. Then the total energy E_T will be deformed to[1]

$$E_T = NE + \frac{\beta_1}{\Lambda} \left(\frac{N^2 - N}{2} \right) E^2 = E_\Sigma + \frac{\beta_1}{\Lambda} \frac{N-1}{2N} \; E_\Sigma^2, \tag{6}$$

where the sum of the contributions $E_\Sigma = \sum_i E_i = NE$ could be, in principle, much larger than the characteristic energy scale of the deformation Λ. Of course this rough description of macroscopic manifestation of the deformed composition law for momenta (soccer-ball problem) is in contradiction with many observations, and should consequently be ruled out. In this paper we will consider a simple way of including a restriction on the non-additivity effects: we recognize this phenomenon to be a coherence process so that it should be confined within a certain *coherence length*.

It is clear that two particles that are not correlated in any way, and far apart from each other, are really independent systems so that the total energy will be just the sum of their energies, with no nonlinear corrections. Therefore one can expect that the modification of the energy composition law will be limited to particles separated by a distance smaller than the coherence length ℓ_c. We do not know the physical origin of this restriction on the non-additive corrections and how this length scale ℓ_c emerges. At the classical level, the possible effective interaction associated to the quantum nature of space–time will take place in a region of Planckian size. This suggests to attribute the energy non-additivity to the quantum "delocalization" of particles, which might lead to a coherence length much larger than the Planck length. In this sense, an interesting possibility is that the non trivial composition law for energies were a consequence of the superposition of different wave functions in a given quantum system; the coherence length would then define the range of this superposition. In any case, it seems clear that this length must be a microscopic scale, because it should provide a solution to the soccer-ball problem, but it may well depend on the properties of the system under consideration, such as the type of particles, whether or not they are entangled, or on their quantum mechanical properties, such as their de Broglie or Compton wavelengths.

Recent results in Lie-algebra noncommutativity models [15] suggest that macroscopic objects are blind to some effects of the quantization of space–time, because of a suppression factor of

[1] To illustrate the point we will use only the β_1-term of Eq. (5).