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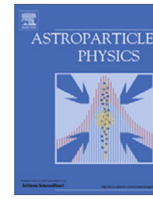

Could pressureless dark matter have pressure?

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ABSTRACT

A two-fluid dark matter model, in which dark matter is represented as a two-component fluid thermodynamic system, without interaction between the constituent particles of different species, and with each distinct component having a different four-velocity, was recently proposed in Harko and Lobo [T. Harko, F.S.N. Lobo, Phys. Rev. D 83 (2011) 124051]. In the present paper we further investigate the two-fluid dark matter model, by assuming that the two dark matter components are pressureless, non-comoving fluids. For this particular choice of equations of state the dark matter distribution can be described as a single anisotropic fluid, with vanishing tangential pressure, and non-zero radial pressure. We investigate the properties of this model in the region of constant velocity galactic rotation curves, where the dynamics of the test particles is essentially determined by the dark matter only. By solving the general relativistic equations of mass continuity and hydrostatic equilibrium we obtain the geometric and physical parameters of the dark matter halos in the constant velocity region in an exact analytical form. The general, radial coordinate dependent, functional relationship between the energy density and the radial pressure is also determined, and it differs from a simple barotropic equation of state.

1. Introduction

The Concordance Cosmological Model, usually referred to as the Λ cold dark matter (ΛCDM) model, has proved to be very successful in explaining cosmological observations across a wide rage of length scales, from the cosmic microwave background (CMB) anisotropy to the Lyman-α forest [1,2]. In this model, nonbaryonic collisionless cold dark matter makes up to 23% of the total mass content of the Universe. In the ΛCDM model, dark matter consists of cold neutral weakly interacting massive particles, beyond those existing in the Standard Model of Particle Physics. However, up to now no dark matter candidates have been detected in particle accelerators or in direct and indirect searches. Many particles have been proposed as possible candidates for dark matter, the most popular ones being the Weakly Interacting Massive Particles (WIMP) and the axions (for a review of the particle physics aspects of dark matter see [3]). The interaction cross section of dark matter particles with normal baryonic matter is assumed to be extremely small. However, it is expected to be non-zero, and therefore the direct experimental detection of dark matter particles may be possible. Superheavy particles, with mass $\geqslant 10^{10}$ GeV, have also been proposed as dark matter candidates. But in this case observational results show that these particles must either interact weakly with normal matter, or they must have masses above 10^{15} GeV [4]. Scalar field models, or other long range coherent fields coupled to gravity have also been proposed to model galactic dark matter [5–21]. The possibility that dark matter could be described by a fluid with non-zero effective pressure was also investigated [22,23]. In particular, it was assumed in [24] that equation of state of the dark matter halos is polytropic. The fit with a polytropic dark halo improves the velocity dispersion profiles. The possibility that the galactic dynamics of massive test particles may be understood without the need for dark matter was explored in the context of modified theories of gravity in [25–32].

On galactic scales observational data seem to disagree with the ΛCDM model predictions. High resolution N-body simulations have shown that the predicted number of subhalos is an order of magnitude larger than what has been observed [33]. Another discrepancy arises when comparing the density profiles of dark halos predicted in simulations with those derived from observations of dwarf spheroidal (dSph) galaxies and Low Surface Brightness galaxies (LSBs). N-body simulations predict an universal cuspy density profile [34,35],

$$\rho_{NFW}(r) \approx \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}, \tag{1}$$

where r_s is a scale radius and ρ_s is a characteristic density. On the other hand observations based on high-resolution rotation curves show, instead, that the actual distribution of dark matter is much

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shallower than the above, thus indicating that a cored halo is preferred in an important fraction of low-mass galaxies [36],

$$\rho_B(r) \approx \frac{\rho_0 r_0^3}{(r+r_0)(r^2+r_0^2)}, \quad (2)$$

where r_0 is the core radius and ρ_0 is the central density. The observational Burkert density profile, given by Eq. (2), resembles an isothermal profile in the inner regions, i.e., $r \ll r_0$, predicts a finite central density, ρ_0, and leads to a mass profile that diverges logarithmically for increasing r, which is consistent with cosmological cold dark matter predictions [34,35].

These discrepancies between theoretical predictions and observations might be overcome by considering other alternative dark matter models. The possibility that dark matter is a mixture of two non-interacting perfect fluids, with different four-velocities and thermodynamic parameters, was proposed recently in [37]. By introducing a rotation of the four-velocity vectors the two-fluid model can be reduced to an effective single anisotropic fluid model, with distinct radial and tangential pressures [38–40]. By assuming a non-relativistic kinetic model for the dark matter particles, the density profile and the tangential velocity of the dark matter mixture have been obtained by numerically integrating the gravitational field equations. The cosmological implications of the model have also been briefly analyzed, and it was shown that the anisotropic two-fluid model isotropizes in the large time limit. Two fluid dust models have also been considered in a general relativistic framework in [41,42].

It is the purpose of the present paper to further investigate the idea proposed in [37], by considering the specific case of two, non-interacting, *pressureless* dark matter fluids. For this configuration the model reduces to a single anisotropic fluid, with vanishing tangential pressure. We investigate the properties of this model in the region of constant galactic velocity rotation curves, where the solution of the basic equations can be obtained in an exact analytical form. Thus, the radial coordinate dependence of all relevant geometric and physical parameters of the dark matter halos is explicitly determined. In particular, we obtain the general, r-dependent, functional relationship between the energy density and the radial pressure of the dark matter, which differs from the simple barotropic equation of state previously considered in the physical literature [22–24].

The present paper is organized as follows. The two-fluid model of the dark matter halos is briefly reviewed in Section 2. The general relativistic structure equations for anisotropic fluids are written down in Section 3, and the tangential velocity of test particles in stable circular orbits is obtained as a function of the geometric metric tensor. The general solution of the gravitational field equations in the constant velocity region of the dark matter halos is obtained in Section 4. We discuss and conclude our results in Section 5. In the present paper we use the natural system of units with $c = G = \hbar = 1$.

2. Dark matter as a mixture of two perfect fluids

We start our study of dark matter by assuming that it consists of a mixture of two perfect fluids, with energy densities and pressures ρ_1, p_1 and ρ_2, p_2, respectively, and with four velocities U^μ and W^μ, respectively. The fluid is described by the total energy–momentum tensor $T^{\mu\nu}$, given by

$$T^{\mu\nu} = (\rho_1+p_1)U^\mu U^\nu - p_1 g^{\mu\nu} + (\rho_2+p_2)W^\mu W^\nu - p_2 g^{\mu\nu}. \quad (3)$$

The four-velocities are normalized according to $U^\mu U_\mu = 1$ and $W^\mu W_\mu = 1$, respectively. The study of the physical systems described by an energy–momentum tensor having the form given by Eq. (3) can be significantly simplified if we cast it into the standard form of perfect anisotropic fluids. This can be done by means of the transformations $U^\mu \to U^{*\mu}$ and $W^\mu \to W^{*\mu}$, respectively, so that [38–40]

$$\begin{pmatrix} U^{*\mu} \\ W^{*\mu} \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sqrt{\frac{\rho_2+p_2}{\rho_1+p_1}}\sin\alpha \\ -\sqrt{\frac{\rho_1+p_1}{\rho_2+p_2}}\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} U^\mu \\ W^\mu \end{pmatrix}, \quad (4)$$

representing a "rotation" of the velocity four-vectors in the (U^μ, W^μ) velocity space. Notice that the transformations given by Eq. (4) leave the quadratic form $(\rho_1+p_1)U^\mu U^\nu + (\rho_2+p_2)W^\mu W^\nu$ invariant. Thus we have

$$T^{\mu\nu}(U,W) = T^{\mu\nu}(U^*,W^*). \quad (5)$$

As for the vectors $U^{*\mu}$ and $W^{*\mu}$ we assume that one is timelike, while the other is spacelike, so that $U^{*\mu}W^*_\mu = 0$.

With the use of the latter relationship and Eq. (4) we obtain the rotation angle as

$$\tan 2\alpha = 2\frac{\sqrt{(\rho_1+p_1)(\rho_2+p_2)}}{\rho_1+p_1-(\rho_2+p_2)}U^\mu W_\mu. \quad (6)$$

Next we define the following quantities [38–40]:

$$V^\mu = \frac{U^{*\mu}}{\sqrt{U^{*\alpha}U^*_\alpha}}, \quad \chi^\mu = \frac{W^{*\mu}}{\sqrt{-W^{*\alpha}W^*_\alpha}}, \quad (7)$$

$$\varepsilon = T^{\mu\nu}V_\mu V_\nu = (\rho_1+p_1)U^{*\alpha}U^*_\alpha - (p_1+p_2), \quad (8)$$

$$\sigma = T^{\mu\nu}\chi_\mu\chi_\nu = (p_1+p_2) - (\rho_2+p_2)W^{*\alpha}W^*_\alpha, \quad (9)$$

$$\Pi = p_1+p_2, \quad (10)$$

respectively. Thus, the energy–momentum tensor of the two non-interacting perfect fluids can be written as

$$T^{\mu\nu} = (\varepsilon+\Pi)V^\mu V^\nu - \Pi g^{\mu\nu} + (\sigma-\Pi)\chi^\mu\chi^\nu, \quad (11)$$

where

$$V^\mu V_\mu = 1 = -\chi^\mu\chi_\mu, \quad (12)$$

and

$$\chi^\mu V_\mu = 0. \quad (13)$$

Note that the energy–momentum tensor given by Eq. (11) is the standard form for anisotropic fluids [40].

The energy density ε and the radial pressure σ are given by

$$\varepsilon = \frac{1}{2}(\rho_1+\rho_2-p_1-p_2) + \frac{1}{2} \times\sqrt{(\rho_1+p_1+\rho_2+p_2)^2 + 4(\rho_1+p_1)(\rho_2+p_2)\left[(U^\mu W_\mu)^2-1\right]}, \quad (14)$$

and

$$\sigma = -\frac{1}{2}(\rho_1+\rho_2-p_1-p_2) + \frac{1}{2} \times\sqrt{(\rho_1+p_1-\rho_2-p_2)^2 + 4(\rho_1+p_1)(\rho_2+p_2)(U^\mu W_\mu)^2}, \quad (15)$$

respectively [38–40].

In comoving spherical coordinates $x^0 = t$, $x^1 = r$, $x^2 = \vartheta$, and $x^3 = \phi$ we may choose $V^1 = V^2 = V^3 = 0$, $V^0V_0 = 1$, and $\chi^0 = \chi^2 = \chi^3 = 0$, $\chi^1\chi_1 = -1$ [38–40]. Therefore the components of the energy–momentum of two non-interacting perfect fluids take the form

$$T_0^0 = \varepsilon, \quad T_1^1 = -\sigma, \quad T_2^2 = T_3^3 = -\Pi, \quad (16)$$

where ε is the total energy–density of the mixture of fluids, $\sigma = P_r$ is the pressure along the radial direction, while $\Pi = P_\perp$ is the tangential pressure on the $r =$ constant surface.