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Mathematical Model for Measuring Pulsar Parameters by Pulsar Timing Observations and Precision Estimation† ⋆

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Abstract Pulsar rotational and astrometric parameters can be measured with a very high precision by pulsar timing observations. The pulsar timing model, the methods to measure the pulsar parameters by least squares fitting and to estimate their covariances are briefly described. The mathematical relations between the timing residuals of a pulsar and the errors of its different parameters are derived in the ecliptic coordinate system. The relationships of the measuring precisions of pulsar's ecliptic longitude, latitude, and annual parallax with the absolute value of pulsar's ecliptic latitude are shown by curves. And the relations of the measuring precisions of pulsar's astrometric parameters with the pulsar's latitudinal parameter itself are discussed as well.

Key words astrometry, reference system, pulsar, methods: analytical

1. INTRODUCTION

The rotational parameters (the rotational phase, rotation frequency and its derivative at a reference epoch) and astrometric parameters (position, proper motion and annual parallax)

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of a pulsar can be measured with a very high precision by pulsar timing observations. A series of pulsars, which are distributed in the different directions on the sky, with their rotational and astrometric parameters accurately measured, make a pulsar space-time reference frame, which is of great importance in practical applications as space-time reference coordinates. The pulsar timing observation of a radio telescope provides the times of arrival (TOA) of pulsar pulses to the telescope, and the pulsar rotational parameters and astrometric parameters may be obtained by analyzing the time series of TOA in the perennial observation on the same pulsar with the same telescope. In this paper, we discuss the mathematical model and measuring precision for the measurements of pulsar parameters by pulsar timing observations.

2. A BRIEF DESCRIPTION OF PULSAR TIMING MODEL

The relation between the rotational phase of a pulsar and its rotational parameter in the pulsar reference frame may be expressed as

$$\phi(t) = \phi_0 + \nu(t - t_0) + \frac{1}{2}\dot{\nu}(t - t_0)^2 \,, \tag{1}$$

in which t and t_0 indicate the pulsar time with t_0 corresponding to the reference epoch; ϕ_0, ν and $\dot{\nu}$, the pulsar rotational phase, rotation frequency and its 1st-order derivative at the time t_0. Let t_p be the pulse time predicted by a pulsar clock, $P(t)$, the pulsar rotational period at the time t, then

$$t_\mathrm{p} = \phi(t)P(t) = [\phi_0 + \nu(t - t_0) + \frac{1}{2}\dot{\nu}(t - t_0)^2]P(t) \,. \tag{2}$$

Assuming that t_obs is the time of arrival (TOA) of the same pulsar pulse observed by the ground-based radio telescope in reference to an atomic clock, the relation between t_obs and t_p may be set up as follows:

$$\begin{aligned} t_\mathrm{p} = {} & t_\mathrm{obs} + \Delta t_\mathrm{c} - \frac{1}{c}[(\hat{\boldsymbol{n}}\cdot\vec{V})\Delta t - (\hat{\boldsymbol{n}}\cdot\vec{r})] - \frac{1}{2cR_0}[\vec{r}^2 - (\hat{\boldsymbol{n}}\cdot\vec{r})^2] + \frac{1}{cR_0}[\vec{V}\cdot\vec{r} - \\ & (\hat{\boldsymbol{n}}\cdot\vec{V})(\hat{\boldsymbol{n}}\cdot\vec{r})]\Delta t - \frac{1}{2cR_0}[\vec{V}^2 - (\hat{\boldsymbol{n}}\cdot\vec{V})^2]\Delta t^2 + \sum_{k=1}^{n}\frac{2GM_k}{c^3}\ln|\hat{\boldsymbol{n}}\cdot\vec{r}_k + r_k| - \\ & \frac{4G^2 m_\odot^2}{c^5 r_\odot \tan\psi \sin\psi} - \Delta t_\mathrm{ob} \,, \end{aligned} \tag{3}$$

in which Δt_c includes the correction from the time of atomic clock as the reference in the timing observation to the terrestrial time TT and the correction from TT to the barycentric coordinates time TCB, and these two corrections are generally obtained by the interpolation on a numerical table calculated in advance[1]. Generally speaking, Δt_c has to include also the correction from TCB to the pulsar time. c is the velocity of light; $\hat{\boldsymbol{n}}$, the unit vector of the pulsar in the barycentric coordinate system (of the solar system); $\vec{V}$, the velocity vector of