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Self-generated magnetic fields in blast-wave driven Rayleigh-Taylor experiments

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ABSTRACT

We study the effect of self-generated magnetic fields in two-dimensional computer models of blast-wave driven high-energy density Rayleigh-Taylor instability (RTI) experiments. Previous works [1,2] suggested that such fields have the potential to influence the RTI morphology and mixing. When neglecting the friction force between electrons and ions, we do indeed find that dynamically important ($\beta \lesssim 10^3$) magnetic fields are generated. However, in the more realistic case where the friction force is accounted for, the resulting fields are much weaker, $\beta \gtrsim 10^5$, and can no longer influence the dynamics of the system. Although we find no evidence for dynamically important magnetic fields being created in the two-dimensional case studied here, the situation might be different in a three-dimensional setup, which will be addressed in a future study.

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1. Introduction

Observational as well as theoretical studies [3] have shown that during a supernova explosion, various fluid-dynamical instabilities (such as the Rayleigh-Taylor instability [RTI,see refs. [4,5]]) lead to mixing between the material shells of the exploding star. Understanding the observed mixing has been proved difficult, and in many cases numerical simulations of supernova explosions do not match observational results [see [6], and references therein].

High-energy density laboratory astrophysics experiments [7,8,9] which reproduce the physical conditions in the exploding star can be used as a tool to help understand the physical processes responsible for the mixing. In these experiments, one uses powerful lasers to drive a shock wave through a target, which is composed of two or more layers of materials with different densities. The mixing that occurs between the material layers can be observed via x-ray radiography. Although there is a huge difference in scale between such experiments and a supernova, a direct connection between the two systems can be made by employing similarity scaling [10], provided that viscous and radiative effects can be neglected.

The first experiments of this type were performed on the NOVA laser [11,12]. Later work by Kuranz and her collaborators [9,13,14] continued on the OMEGA laser using planar targets. In one particular case [14], the experimental radiographic images indicated the presence of dense plasma protruding from the tips of the RTI spikes toward the shock front. Subsequent computational studies of the experiment consistently failed to reproduce these features [9,15,16]. It has been suggested [1] that the reason for the discrepancy between experiment and simulations might be due to magnetic fields generated by the Biermann battery effect, which was not included in the numerical simulations.

Self-generated magnetic fields have been measured in an ablative RTI experiments, both during the linear [17] and non-linear [18] growth phase of the instability. Numerical simulations indicate that magnetic fields can be dynamically important in the context of inertially confined fusion [2].

The aim of the present study is to investigate to what extent self-generated magnetic fields might have influenced the results of the Kuranz et al. experiment and if such fields might be important in other (similar) experiments. We discuss the results of numerical simulations of blast-wave driven RTI experiments based on the extended magnetohydrodynamics (MHD) formulation according to Braginskii [19], which includes the Biermann battery term. We consider two sets of parameters, where the first set corresponds to the Kuranz et al. experiment, and the second set corresponds to a recently proposed experiment [6] for the National Ignition Facility (NIF) laser.

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2. Model

2.1. Equations solved

For the numerical simulations presented in this paper, we use the PROTEUS code [15]. PROTEUS is an extended version of the FLASH code [20], which includes additional physics for the modelling of anisotropic thermal conduction and magnetic field generation. Our work builds on the study done by Refs. [15], which considered the classical Rayleigh-Taylor problem of a heavy fluid sitting atop of a light fluid in a gravitational field. While their study was done in the context of the Braginskii formulation, it neglected the friction force between electrons and ions. In the present work, we consider blast-wave driven systems with parameters relevant to laser-driven shock experiments and also take into account the dissipation of the magnetic field due to the electron-ion friction force.

We consider the following set of extended MHD equations in the single fluid approximation:

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\boldsymbol{v})=0, \tag{1a}$$

$$\frac{\partial\rho\boldsymbol{v}}{\partial t}+\nabla\cdot\left(\rho\boldsymbol{v}\boldsymbol{v}-\frac{\boldsymbol{B}\boldsymbol{B}}{4\pi}\right)+\nabla P=0, \tag{1b}$$

$$\frac{\partial E}{\partial t}+\nabla\cdot\left[(E+P)\boldsymbol{v}-\frac{\boldsymbol{B}\boldsymbol{B}}{4\pi}\cdot\boldsymbol{v}+\frac{\boldsymbol{B}}{4\pi}\times\boldsymbol{S}_{\mathrm{GenOhm}}\right]=\nabla\cdot\boldsymbol{q}, \tag{1c}$$

$$\frac{\partial\boldsymbol{B}}{\partial t}=\nabla\times(\boldsymbol{v}\times\boldsymbol{B}+\boldsymbol{S}_{\mathrm{GenOhm}}); \tag{1d}$$

where

$$\boldsymbol{S}_{\mathrm{GenOhm}}=\frac{c}{en_{\mathrm{e}}}\left[k_{\mathrm{B}}\nabla(n_{\mathrm{e}}T)-\frac{(\nabla\times\boldsymbol{B})\times\boldsymbol{B}}{4\pi}-\boldsymbol{R}_u-\boldsymbol{R}_T\right]. \tag{1e}$$

In the above equations, ρ denotes the plasma density, $\boldsymbol{v}$ the velocity, E the total energy density, P the total pressure, $\boldsymbol{B}$ the magnetic field, n_{e} the number density of the electrons and T the temperature, where we assume that ion and electron temperatures are equal ($T=T_{\mathrm{e}}=T_{\mathrm{i}}$). The above equations are supplemented by an ideal gas equation of state with adiabatic index $\gamma=5/3$.

We assume a plasma composed of multiple species. The ion number density is $n_{\mathrm{i}}=\rho N_{\mathrm{A}}/A$, with $\overline{A}$ the average atomic mass and N_{A} the Avogadro constant. In order to calculate the electron number density n_{e}, we first compute the (average) plasma charge, $\overline{Z}$, using the Thomas–Fermi equation of state [21]. The corresponding electron number density is then $n_{\mathrm{e}}=\overline{Z}n_{\mathrm{i}}$.

The term on the right hand side of Eq. (1c) describes anisotropic thermal conduction. The heat flux vector $\boldsymbol{q}$ is given by:

$$\boldsymbol{q}=-\kappa_1(\boldsymbol{b}\cdot\nabla T)\boldsymbol{b}-\kappa_2\boldsymbol{b}\times(\nabla T\times\boldsymbol{b})-\kappa_3\boldsymbol{b}\times\nabla T, \tag{2}$$

where $\boldsymbol{b}=\boldsymbol{B}/B$ is a unit vector in the direction of the magnetic field. The coefficients κ_i are of order $n_{\mathrm{e}}k_{\mathrm{B}}T\tau_{\mathrm{e}}/m_{\mathrm{e}}$, where τ_{e} is the electron-ion scattering time [19]. They are related to the coefficients calculated in Braginskii's paper [19] as follows:

$$\kappa_1=\kappa_{\parallel}^{\mathrm{e}}+\kappa_{\parallel}^{\mathrm{i}}, \tag{3a}$$

$$\kappa_2=\kappa_{\perp}^{\mathrm{e}}+\kappa_{\perp}^{\mathrm{i}}, \tag{3b}$$

$$\kappa_3=\kappa_{\wedge}^{\mathrm{e}}-\kappa_{\wedge}^{\mathrm{i}}. \tag{3c}$$

In Eq. (1e), the last two terms in the square brackets are the electron-ion friction force ($\boldsymbol{R}_u$), and the thermal force ($\boldsymbol{R}_T$), which are responsible for diffusing (and thereby destroying) the magnetic field. These two terms are defined by the following formulae:

$$\boldsymbol{R}_u=\alpha_1(\boldsymbol{b}\cdot\boldsymbol{J})\boldsymbol{b}+\alpha_2\boldsymbol{b}\times(\boldsymbol{J}\times\boldsymbol{b})-\alpha_3\boldsymbol{b}\times\boldsymbol{J}, \tag{4}$$

$$\boldsymbol{R}_T=-\beta_1(\boldsymbol{b}\cdot\nabla T)\boldsymbol{b}-\beta_2\boldsymbol{b}\times(\nabla T\times\boldsymbol{b})-\beta_3\boldsymbol{b}\times\nabla T; \tag{5}$$

where $\boldsymbol{J}=(c/4\pi)\nabla\times\boldsymbol{B}$ is the current. The coefficients α_i and β_i are of order $m_{\mathrm{e}}/e\tau_{\mathrm{e}}$ and $k_{\mathrm{B}}n_{\mathrm{e}}$, respectively [for details,see [19]]. The first term inside the square brackets in Eq. (1e) is responsible for the Biermann battery effect. The change in the magnetic field due to this term can be written as

$$\frac{\partial\boldsymbol{B}}{\partial t}=\frac{ck_{\mathrm{B}}}{e}\nabla T_{\mathrm{e}}\times\nabla\ln n_{\mathrm{e}}; \tag{6}$$

i.e. magnetic fields are generated due to a thermally driven current whenever the electron temperature and electron density gradients are misaligned.

For the derivation of the above equations and further details, we point the reader to the original work by Braginskii [19].

In our simulations, we employ the unsplit staggered mesh scheme described in Refs. [22], which uses a constrained transport method that preserves the $\nabla\cdot\boldsymbol{B}=0$ constraint to machine accuracy. For all simulations, we use the Roe Riemann solver and second order reconstruction. We do not consider time-step limiting due to whistler and Hall drift waves. Instead we set the CFL number to a small value (~0.01) to ensure stability.

2.2. Initial conditions

We perform two-dimensional Cartesian simulations of a single blast-wave driven Rayleigh-Taylor spike. For simplicity, our simulations do not include the heating due to the laser drive; instead, the simulations are initialized with a small region of very high pressure on the side of the target which in an experiment would be illuminated by the laser. The domain covers a rectangular region with dimensions $[-L_x/2, L_x/2]\times[0, L_y]$, where L_x and L_y correspond to the lateral and longitudinal domain size, respectively. The initial conditions are as follows:

$$\rho(x,y)=\begin{cases}\rho_{\mathrm{abl}}, & y\le L_{\mathrm{abl}}\\ \rho_{\mathrm{heav}}, & L_{\mathrm{abl}}\le y\le L_{\mathrm{abl}}+L_{\mathrm{heav}}+\delta(x)\\ \rho_{\mathrm{ligh}}, & \text{otherwise}\end{cases}, \tag{7a}$$

$$P(x,y)=\begin{cases}P_{\mathrm{drv}}, & y\le L_{\mathrm{drv}}\\ P_{\mathrm{amb}}, & \text{otherwise}\end{cases}, \tag{7b}$$

$$\boldsymbol{v}(x,y)=0, \tag{7c}$$

$$\boldsymbol{B}(x,y)=\boldsymbol{B}_0. \tag{7d}$$

Here, ρ_{abl}, ρ_{heav} and ρ_{ligh} are the ablator, heavy material and light material densities, respectively. The lengths L_{abl}, L_{heav} and L_{ligh} correspond to the widths of the ablator, heavy material layer and light material layer, respectively. The width of the high pressure region is denoted by Ref. L_{drv} and is set to $L_{\mathrm{drv}}=40\mu$ m in all simulations. The pressure P_{amb} is the ambient pressure and P_{drv} is the drive pressure. The perturbation at the heavy/light