High Energy Density Physics 17 (2015) 114–118

Contents lists available at ScienceDirect

High Energy Density Physics

journal homepage: www.elsevier.com/locate/hedp

Anti-diffusive-like-behavior in semi-analytic radiative shocks via multigroup S_n transport with constant cross sections

A.M. Holgado [a], J.M. Ferguson [b, *], R.G. McClarren [c]

[a] Department of Physics and Astronomy, Texas A & M University, College Station, TX 77843, USA
[b] X-Theoretical Division (XTD), Los Alamos National Laboratory, Los Alamos, NM 87545, USA
[c] Department of Nuclear Engineering, Texas A & M University, College Station, TX 77843, USA

A R T I C L E I N F O

Article history:
Available online 17 March 2015

Keywords:
Radiative shock
Semi-analytic
Multigroup
Planck function
Anti-diffusive

A B S T R A C T

Semi-analytic lab-frame radiative shock solutions have been presented recently, wherein the radiation is modeled with either grey (frequency independent) nonequilibrium-diffusion or grey S_n-transport. As a first step toward incorporating frequency dependence, we retain grey cross-sections such that the frequency dependence of the problem is strictly due to the Planck function. By using multigroup frequency integration the frequency-dependent radiation-transport equation may be solved, and group-dependent radiation temperatures may be defined for each group. Our main result is: When a Zel'dovich temperature spike exists in a radiative shock solution, there exists a transition group g_T, such that for all frequency groups below g_T the group-dependent radiation temperatures are spatially monotonic, and for all frequency groups above g_T the group-dependent radiation temperatures are spatially non-monotonic. We present numerical evidence of our claim and make no claim as to the monotonicity of group g_T.

1. Introduction

The interest to solve frequency-dependent radiative-shock problems has existed in the astrophysics community for decades [1–6]. As computational methods and abilities continue to evolve, it remains pertinent to focus on an understanding of the fundamental physical elements of the problem by either analytic or semi-analytic methods [7–11], which may also serve as code-verification tests, but these works have neglected frequency dependence, and many are based on simplified radiation-diffusion models. The full transport theory of radiation defines the radiation flux as being proportional to the negative gradient of the radiation pressure, but Fick's Law of Diffusion for radiation defines the radiation flux as being proportional to the negative gradient of the radiation energy density; the variable Eddington factor defines the proportional relation between the radiation pressure and the radiation energy density, and in diffusion models is typically assumed to be 1/3 everywhere (the P_1 or Eddington approximation). McClarren and Drake [10], analytically solved the grey comoving-frame radiation-transport equation for a simplified radiative-shock structure, and defined anti-diffusion as occurring when the spatial derivatives of the radiation pressure and the radiation energy densities have different signs, which signals when diffusion models will give a distinctly incorrect result, hence “anti-diffusion”. Previous semi-analytic radiative-shock solutions have been presented by Lowrie and Rauenzahn [8] for frequency-independent (grey) equilibrium-diffusion, by Lowrie and Edwards [9] for grey nonequilibrium-diffusion, and Ferguson, Morel, and Lowrie [11] for grey S_n-transport. The S_n-transport method allows the variable Eddington factor (VEF) to be determined; in one dimension S_2 is equivalent to diffusion and also setting the VEF to 1/3. The latter work confirmed the existence of anti-diffusion, and that it is associated with a local maximum in the radiation energy density, and also when the variable Eddington factor takes values less than 1/3.

We extend the work of Ferguson, Morel and Lowrie [11] to solve the frequency-dependent lab-frame S_n radiation-transport equation by multigroup integrating the frequency domain. Multigroup frequency integration decomposes the frequency domain into smaller groups, which when summed over recreate the complete frequency domain. This allows us to investigate whether group-dependent radiation energy densities exhibit local maxima, for both diffusion and S_n-transport models ($n > 2$). When the radiation energy density exhibits anti-diffusion, from an S_n-transport solution, we show that some of the group-dependent radiation energy densities also exhibit anti-diffusion. While the radiation energy

* Corresponding author.
E-mail address: jmferguson@lanl.gov (J.M. Ferguson).

http://dx.doi.org/10.1016/j.hedp.2015.03.004
1574-1818/

density from a diffusion solution might not exhibit anti-diffusion, we also show that some of the group-dependent radiation energy densities appear to exhibit a local maximum. Since these local maximum are not associated with anti-diffusion of the radiation energy density, we label it "anti-diffusive-like-behavior" (ADLB). By solving the frequency-dependent radiation-transport equation, but restricting ourselves to grey material cross sections, we are able to explain the ADLB of the group-dependent radiation energy densities in terms of the multigroup frequency-integrated Planck function. This work represents the initial step to incorporate frequency dependence into semi-analytic radiative-shock solutions.

2. Governing equations

In this section we present the steady-state, nondimensional equations of radiation transport and radiation hydrodynamics. We follow the nondimensionalization presented by Lowrie and Edwards [9] and Ferguson, Morel and Lowrie [11] throughout; variables containing a tilde-˜ carry the appropriate dimension. We add frequency-dependent variables to their non-dimensionalization, present the frequency-dependent radiation-transport equation and introduce multigroup frequency-integration of this equation and the frequency-dependent radiation variables. The frequency-integrated (grey) equation and variables are used to define the radiation variables, and derive the radiation sources, that are used in the conservation equations of radiation hydrodynamics. In steady-state, these conservation equations reduce to a system of two ordinary differential equations that can be solved to produce radiative-shock solutions.

2.1. Frequency-dependent nondimensionalization

We add to the nondimensional definitions of Ferguson, Morel and Lowrie [11] the nondimensional definitions of frequency, the frequency-dependent radiation intensity, the frequency-dependent Planck function, and we include the material temperature, respectively:

$$\nu = \tilde{\nu}\frac{\tilde{h}}{\tilde{k}_B\tilde{T}_0}, \tag{1a}$$

$$I_\nu = \tilde{I}_{\tilde{\nu}}\frac{\tilde{k}_B}{\tilde{\alpha}_R\,\tilde{c}\,\tilde{h}\,\tilde{T}_0^3}, \tag{1b}$$

$$B_\nu = \tilde{B}_{\tilde{\nu}}\frac{\tilde{k}_B}{\tilde{\alpha}_R\,\tilde{c}\,\tilde{h}\,\tilde{T}_0^3}, \tag{1c}$$

$$T = \frac{\tilde{T}}{\tilde{T}_0}, \tag{1d}$$

where ν is the frequency, I_ν is the frequency-dependent radiation intensity, B_ν is Planck's function, $\tilde{h}$ is Planck's constant, $\tilde{k}_B$ is Boltzmann's constant, $\tilde{\alpha}_R$ is the radiation constant, $\tilde{c}$ is the speed of light, $\tilde{T}_0$ is a reference temperature which takes its value at the upstream equilibrium pre-shock state, $\tilde{T}$ is the material temperature of the fluid, and T is the nondimensional material temperature. This nondimensionalization (1) was chosen so that I_ν and B_ν would have the same nondimensionalization, just as $\tilde{I}_{\tilde{\nu}}$ and $\tilde{B}_{\tilde{\nu}}$ have the same dimensions, and so that the nondimensional Planck function, determined by expressions (1a) and (1c),

$$B_\nu = \frac{2\tilde{k}_B^4}{\tilde{\alpha}_R\tilde{c}^3\tilde{h}^3}\frac{\nu^3}{e^{\nu/T}-1}, \tag{2}$$

when frequency integrated would produce a sensible nondimensional expression,

$$\int_0^\infty B_\nu\,d\nu = \frac{1}{\tilde{\alpha}_R\tilde{c}\tilde{T}_0^4}\int_0^\infty \tilde{B}_{\tilde{\nu}}\,d\tilde{\nu} = \frac{1}{4\pi}\frac{\tilde{\alpha}_R\tilde{c}\tilde{T}^4}{\tilde{\alpha}_R\tilde{c}\tilde{T}_0^4} = \frac{T^4}{4\pi}. \tag{3}$$

Setting the derivative of the nondimensional Planck function with respect to the nondimensional frequency to zero produces an expression relating the nondimensional peak frequency to the nondimensional material temperature,

$$\nu_{\text{peak}} = 2.821\,T, \tag{4a}$$

which agrees with the standard expression relating the peak frequency to the material temperature

$$\tilde{\nu}_{\text{peak}} = 2.821\,\frac{\tilde{k}_B\tilde{T}}{\tilde{h}}, \tag{4b}$$

after using the nondimensionalizations for frequency (1a) and temperature (1d). The nondimensional Planck function admits a functional form of the Rayleigh-Jeans Law for $\nu \ll \nu_{\text{peak}}$,

$$B_\nu = \frac{2\tilde{k}_B^4}{\tilde{\alpha}_R\tilde{c}^3\tilde{h}^3}\nu^2 T, \tag{5a}$$

and Wien's Law for $\nu \gg \nu_{\text{peak}}$,

$$B_\nu = \frac{2\tilde{k}_B^4}{\tilde{\alpha}_R\tilde{c}^3\tilde{h}^3}\nu^3 e^{-\nu/T}, \tag{5b}$$

which will define our frequency boundaries.

The multigroup method typically bounds the semi-infinite frequency domain $\nu \in [0,\infty)$ into a smaller subset $\nu \in [\nu_{\min}, \nu_{\max}]$, and discretizes this subset into G finite groups so that $g \in \mathbb{Z}[0, G-1]$ represents one group whose frequency domain is $\nu \in [\nu_g, \nu_{g+1}]$ such that $\cup_g[\nu_g, \nu_{g+1}] = [\nu_{\min}, \nu_{\max}]$. We choose to further impose that $\nu_{\min}$ and $\nu_{\max}$ should be determined via the Rayleigh-Jeans Law (5a) and Wien's Law (5b), respectively, such that $B_{\nu\min} = 10^{-20} = B_{\nu\max}$, in order to retain the correct shape of the Planck function. The value 10^{-20} was chosen so that the resulting values of $\nu_{\min}$ and $\nu_{\max}$ would reasonably estimate the frequency-integrated Planck function

$$\int_{\nu\min}^{\nu_{\max}} B_\nu\,d\nu \approx \int_0^\infty B_\nu\,d\nu. \tag{6}$$

Of course, the values for $\nu_{\min}$ and $\nu_{\max}$ may be significantly shifted higher or lower without changing the integrated value of the nondimensional Planck function, but this will distort the shape of the Planck function. While this may not be expected to cause a problem, we show evidence in Section 3 that the shape of the Planck function may control the shape of the radiative shock, especially the radiation temperature near the Zel'dovich spike.