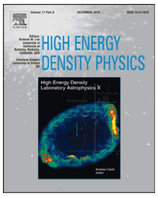
High Energy Density Physics 17 (2015) 135–139

Contents lists available at ScienceDirect

High Energy Density Physics

journal homepage: www.elsevier.com/locate/hedp



Numerical simulations of Mach stem formation via intersecting bow shocks

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ARTICLE INFO

Article history:
Available online 17 January 2015

Keywords:
Shock wave phenomena
Herbig-Haro objects
ISM
Mach stems
Jets and outflows

ABSTRACT

Hubble Space Telescope observations show bright knots of Hα emission within outflowing young stellar jets. Velocity variations in the flow create secondary bow shocks that may intersect and lead to enhanced emission. When the bow shocks intersect at or above a certain critical angle, a planar shock called a Mach stem is formed. These shocks could produce brighter Hα emission since the incoming flow to the Mach stem is parallel to the shock normal. In this paper we report first results of a study using 2-D numerical simulations designed to explore Mach stem formation at the intersection of bow shocks formed by hypersonic "bullets" or "clumps". Our 2-D simulations show how the bow shock shapes and intersection angles change as the adiabatic index γ changes. We show that the formation or lack of a Mach stem in our simulations is consistent with the steady-state Mach stem formation theory. Our ultimate goal, which is part of an ongoing research effort, is to characterize the physical and observational consequences of bow shock intersections including the formation of Mach stems.

1. Introduction

Astrophysical jets are heterogeneous beams of plasma traveling at supersonic velocities. They can be found in a variety of environments at different scales (see Ref. [1] for an overview). Herbig-Haro (HH) jets are associated with young stellar objects (YSOs) and the star formation process. They propagate away from YSOs and interact with the interstellar medium. HH jets are ubiquitous in star forming regions because they are, most likely, closely correlated to the accretion processes creating those stars. Thus it follows that time variability in the accreting disk produces variability in the outflowing jet [2,3].

Structures within jet beams can be caused by variations in the momentum injection at the jet source [4]. This variability results in a "clumpy" jet flow leading to a variety of heterogeneous interactions along the jet beam (see e.g. Refs. [5–7]). Hubble Space Telescope (HST) time-series observations of HH objects reveal localized bright knots in Hα and regions of strong [S II] [8]. Some of these Hα features represent shock fronts caused by variable velocities, and the [S II] regions represent cooling regions behind shocks [9]. Many groups have studied jet models with a variable injection velocity (e.g. Refs. [10,11]). Recent high resolution MHD simulations by Hansen et al. [12] explored how these variable velocity jet models produce internal shocks and the Hα and [S II] emission patterns they produce.

The HST observations also reveal that some of these Hα knots are brighter than expected, and these are located at the intersection points between separate bow shocks. There is still some uncertainty as to why these knots are brighter, but one possible explanation is that a Mach stem formed at the intersection. When bow shocks intersect at an angle at or above a certain critical value, a third shock (Mach stem) will form. Mach stems form perpendicular to the direction of flow, so incoming plasma will encounter a planar shock instead of an oblique one. A planar shock would theoretically lead to brighter emission at this location. In Ref. [8], Fig. 7 shows HST images of a region of HH 2 with Hα in green and [S II] in red. This region is an ideal laboratory for studying potential Mach stems as it consists of many clumps and small secondary bow shocks.

High Energy Density Laboratory Experiments have also been conducted in order to understand Mach stems [13,14]. These experiments were also important to the efforts surrounding inertial-confinement fusion because they are thought to be related to various plasma instabilities such as the Kelvin-Helmholtz instability [15]. By creating oblique shocks running past a reflecting wall

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http://dx.doi.org/10.1016/j.hedp.2014.12.005
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these experiments created situations that are analogous to the astrophysical scenario of two intersecting bow shocks. The main goal of these experiments was to explore the Mach stem growth rate as a function of angle between the shock flow and the obstructing wall.

In this work, we focus on the astrophysical scenario by conducting a set of high resolution simulations of intersecting bow shocks in 2-D. We explore how the ratio of specific heats γ affects the shape of the bow shock and whether or not this allows a Mach stem to form given the separation d of clumps producing the bow shocks. We show that our simulations are in good agreement with the theory of Mach stem formation, and in future work, we will explore other properties and consequences of Mach stems, such as emission, more thoroughly.

The structure of the paper is as follows: in Section 2, we give a brief overview of the theory of Mach stem formation. In Section 3, we present some of our numerical methods as well as our initial conditions for the simulations. Section 4 contains our simulation results and our interpretations. Finally, we finish the paper with our conclusions in Section 5.

2. Theory

A description of Mach stem formation, also known as Mach reflection, can be found in several texts (e.g. Refs. [16–18]), so it will suffice to give a brief explanation here. When two bow shocks intersect, they reflect off of each other forming symmetric reflected oblique shocks; this is known as regular reflection. Without a Mach stem, a gas parcel passes through the incident bow shock and then through the reflected shock. The gas is deflected by the shocks in such a way that its final velocity is in the same direction as its initial pre-shock velocity. Once the intersection of the bow shocks reaches a critical angle, determined by the jump conditions for the colliding shocks, a gas parcel can no longer pass through both shocks and maintain its original direction of flow. Therefore, a planar shock known as a Mach stem is formed so that the gas can continue to flow downstream in the same direction as the initial pre-shock flow. The Mach stem extends from each bow shock at positions known as triple points which represent the new positions where the bow shocks reflect (see Fig. 1).

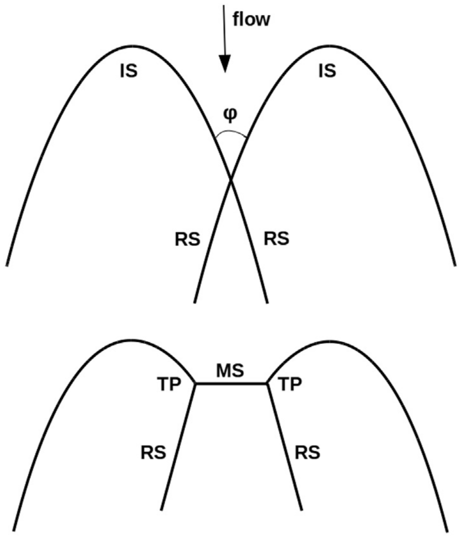

Fig. 1. Diagram of intersecting bow shocks with (*bottom*) and without (*top*) a Mach stem. The acronyms are as follows: “IS” = incident shock (bow shock), “RS” = reflected shock, “MS” = Mach stem, “TP” = triple point. Also shown is the direction of the flow and the included angle φ.

For strong shocks, the critical angle φ_c is only dependent on the ratio of specific heats γ. A simple derivation of φ_c leads to the following approximate formula [17]:

$$\varphi_c = 2\arcsin\left(\frac{1}{\gamma}\right). \tag{1}$$

This approximation breaks down as γ decreases, and it should be noted that the bow shock may be unstable to the Vishniac instability for $\gamma < 1.2$ [19]. A more detailed derivation of this critical angle was done by De Rosa et al. [20] and gives a more accurate equation.

$$\varphi_c = 2\arctan\left[\frac{1}{\gamma-1}\sqrt{\frac{\sqrt{\gamma^2-1}}{\gamma-\sqrt{\gamma^2-1}}}\left(1-\sqrt{\gamma^2-\gamma\sqrt{\gamma^2-1}}\right)\right]. \tag{2}$$

The values of γ used in our simulations result in critical angles shown in Table 1 (in degrees). The critical angle is a *minimum*, so an intersection that occurs at or above this angle should form a Mach stem. Note that as γ decreases, φ_c increases. As γ approaches 1, in other words as the gas becomes isothermal, the Mach stem should not form.

3. Numerical simulations

3.1. Methods

The simulations were carried out using AstroBEAR, a highly parallelized adaptive mesh refinement (AMR) multi-physics code. See Refs. [21,22] for a detailed explanation of how AMR is implemented. More details of the code can also be found at http://bearclaw.pas.rochester.edu/trac/astrobear. Here we provide a brief overview of the physics implemented for this study. The code solves the 2-D Euler equations of fluid dynamics:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\rho\boldsymbol{v} = 0, \tag{3a}$$

$$\frac{\partial\rho v}{\partial t} + \nabla\cdot(\rho vv + P\boldsymbol{I}) = 0, \tag{3b}$$

$$\frac{\partial E}{\partial t} + \nabla\cdot((E+P)v) = 0, \tag{3c}$$

where ρ is the mass density, v is the velocity, P is the thermal pressure, $\boldsymbol{I}$ is the identity matrix, and E is the total energy such that $E = \frac{1}{\gamma-1}P + \frac{1}{2}\rho v^2$ (with $\gamma = \frac{5}{3}$ for an ideal gas). The equations above represent the conservation of mass (3a), momentum (3b), and energy (3c).

3.2. Initial conditions

The simulations consist of 2 stationary clumps in an ambient medium with a wind sweeping over them from the top boundary. In 2-D, these clumps are cross sections of cylinders. This wind

Table 1
Critical angles φ_c (in degrees) for selected values of γ.

γ	φ_c
5/3	74.75
1.4	83.31
1.2	95.34
1.01	137.46