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The viscosity to entropy ratio: From string theory motivated bounds to warm dense matter transport

G. Faussurier [a,*], S.B. Libby [b], P.L. Silvestrelli [c,d]

[a] CEA, DAM, DIF, F-91297 Arpajon, France
[b] Physics Division, Physical and Life Sciences, Lawrence Livermore National Laboratory, P.O. Box 808, Livermore, CA 94550, USA
[c] Dipartimento di Fisica e Astronomia, Università di Padova, via Marzolo 8, I-35131 Padova, Italy
[d] DEMOCRITOS National Simulation Center, Istituto Officina dei Materiali, Consiglio Nazionale delle Ricerche, Trieste, Italy

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ABSTRACT

We study the ratio of viscosity to entropy density in Yukawa one-component plasmas as a function of coupling parameter at fixed screening, and in realistic warm dense matter models as a function of temperature at fixed density. In these two situations, the ratio is minimized for values of the coupling parameters that depend on screening, and for temperatures that in turn depend on density and material. In this context, we also examine Rosenfeld arguments relating transport coefficients to excess reduced entropy for Yukawa one-component plasmas. For these cases we show that this ratio is always above the lower-bound conjecture derived from string theory ideas.

1. Introduction

In 2005, Kovtun et al. (KSS) [1] conjectured from string theory arguments that in general, equilibrium media,

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}, \tag{1}$$

where η is the shear viscosity, s the entropy density, $\hbar$ the reduced Planck constant, and k_B the Boltzmann constant. This inequality is supposed to be valid for any finite-temperature relativistic quantum-field theory with zero chemical potential. Equality is obtained for theories with gravity duals, i.e., with anti de Sitter(-AdS)/conformal field theory (CFT) correspondence. Given the speed of light does not appear, the inequality (1) could be true for non-relativistic systems. Kovtun et al. [1] further motivated their conjecture by showing data proving that (1) holds for common substances such as helium, nitrogen, or water. The inequality (1) can be examined for more exotic states of matter such as cold atomic gases or hot quark gluon plasmas (QGP) [2]. Indeed, the AdS/CFT correspondence finds applications in relativistic heavy-ion collisions in which QGP can be produced [3].

Thoma and Morfill [4] studied what happens to (1) in one-component plasmas (OCP) [5–8]. They have shown that inequality (1) is also valid for OCP, demonstrating that the ratio η/s is indeed minimized as function of the coupling parameter in strongly-coupled OCP when the thermal de Broglie length Λ is kept equal to the Wigner–Seitz radius d.

In this work, we extend the study of Thoma and Morfill [4] in two directions, to the case of the Yukawa screened OCP (YOCP) [8–16]. We also consider more realistic models of solid density materials in the warm dense matter (WDM) regime. We discuss the relevance of the Rosenfeld quasi-universal scaling law relating transport coefficients to excess reduced entropy in simple fluids [17,18]. The aim of this paper is to see what happens to the viscosity to entropy-density ratio in warm dense matter or screened strongly-coupled one-component plasmas.

2. YOCP

The YOCP are characterized by two parameters [16], i.e., the coupling parameter

$$\Gamma = \frac{(Ze)^2}{k_B T d} \tag{2}$$

and the screening parameter

$$\kappa = d/\lambda_D. \tag{3}$$

* Corresponding author.
E-mail address: gerald.faussurier@cea.fr (G. Faussurier).

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In these expressions, Z is the particle charge in units of elementary charge e, T the temperature, k_B the Boltzmann constant, and λ_D the screening-length. This model considers a single type of particle of charge Ze in a polarizable charge-neutralizing background for which the screening property of the plasma is expressed by the Yukawa potential

$$\phi(R) = \frac{(Ze)e^{-R/\lambda_D}}{R}. \quad (4)$$

λ_D can be described by the Debye length, or more generally, by the finite-temperature Thomas–Fermi screening-length [19–21] which interpolates between the zero-temperature Thomas–Fermi screening-length and the Debye length.

The thermal de Broglie length $\Lambda = \sqrt{2\pi\hbar^2/mk_BT}$, where m is the mass of the plasma particles. The Wigner–Seitz radius d is related to the particle number density $n = N/V = 3/4\pi d^3$, where N is the particle number in the volume V. The plasma frequency $\omega_p = \sqrt{4\pi Z^2e^2n/m}$.

The ratio (1) involves two quantities, the shear viscosity η and the entropy density s. In our analysis, we usually consider a dimensionless viscosity

$$\eta^* = \frac{\eta}{mn\omega_p d^2}. \quad (5)$$

Care should be taken, given the various ways dimensionless viscosities appear in the literature. Fits exist for η^* as a function of Γ, κ that are deduced from molecular dynamic (MD) simulations [4,8]. One can show that $m\omega_p d^2 = \sqrt{3\Gamma}v_T dm$, where $v_T = \sqrt{k_BT/m}$ is the thermal velocity of the plasma particles. By definition, the entropy density $s = S/V$. In YOCP, we usually consider the dimensionless internal energy $u = U/Nk_BT$, free energy $f = F/Nk_BT$, and entropy $s^* = S/Nk_B$. U, F, and S are the internal energy, free energy, and entropy related by the thermodynamic relation $F = U - TS$ of the particle system. We consider only systems in thermodynamic equilibrium. Anticipating our application of the Rosenfeld scaling method, the normalized entropy s^* can be split into two parts, i.e., an ideal part s_{id}, and an excess part s_{ex}

$$s^* = s_{\mathrm{id}} + s_{\mathrm{ex}}. \quad (6)$$

The ideal part is given by Ref. [5]

$$s_{\mathrm{id}} = \frac{5}{2} - \log n - 3\log\Lambda = \frac{5}{2} + \log\left(\frac{4\pi}{3}\right) - 3\log\left(\frac{\Lambda}{d}\right). \quad (7)$$

One can obtain s_{ex} as a function of Γ and κ using fits from MD simulations [9–13] or using the variational modified hypernetted chain (VMHNC) approach [22] applied to the YOCP [15]. One can show [4] that

$$k_B\frac{\eta}{s} = R(\Gamma,\kappa)v_T md, \quad (8)$$

where

$$R(\Gamma,\kappa) = \frac{\sqrt{3\Gamma}\eta^*}{s^*}. \quad (9)$$

Written in the form (8), one can see that $k_B\eta/s$ has the dimensions of an action, since $R(\Gamma,\kappa)$ is dimensionless. Indeed, if we express v_T and d in atomic units, one finds that $k_B\eta/s$ is proportional to $\hbar$. Then, the question concerns the value of this proportionality coefficient with respect to the string theory conjecture. It clearly depends on the system of interest. Note also that η^* and s^* depend on Γ and κ. Since we are considering a classical system of particles, $\Lambda < d$ leading to $v_T md > \sqrt{2\pi}\hbar$. If we use the lower limit that corresponds to $d = \Lambda$ in (8), the string theory limit (1) now reads

$$\sqrt{32\pi^3}R(\Gamma,\kappa) > 1. \quad (10)$$

We plot in Fig 1 the minimum of the left hand side of (10) as a function of Γ for various κ using the VMHNC [15], the Caillol–DeWitt [13,23], or the Hamaguchi [10,11,15] equations of state with a κ-dependent normalized-viscosity fit [8] or the VMHNC with a κ-independent normalized-viscosity fit [8]. We plot also results obtained using the hard-sphere Gibbs-Bogolyubov inequality (HS-GBI) [14]. When the fit of the normalized, i.e., dimensionless viscosity depends on the screening parameter κ (see Table IV in Ref. [8]), results does not depend on the equation of state used. However, we can see that using the fit independent of κ with $\kappa \leq 3$ can have a strong impact on results. This kind of fit should be used with caution. In any case, one can see that inequality (10) is obeyed. Our results are consistent with the one obtained by Thoma and Morfill [4] for OCP. They found 4.89 compared to values around 5. To be complete, we have also plotted results using the variational method based on the Gibbs-Bogolyubov inequality and the hard-sphere system to describe the YOCP [14]. The minimum ratio is a factor two above the previous results. This difference could be explained both by the approximate treatment of the equation of state using the Carnahan–Staring and Perkus–Yevick approximations and by the approximate estimation of the viscosity of OCP using the hard-sphere viscosity [14,16]. Note that the results are smooth and close to 10. Interestingly, this value is close to the values KSS show for helium, nitrogen, and water. Since in this example, we are on the boundary for which quantum effects may play a role, we are not so far from the string theory lower-bound equal to one in our writing. This is one example of a situation for which the value of the minimum ratio is closer to the conjectured lower-bound with the possible exception of the QGP [24]. For

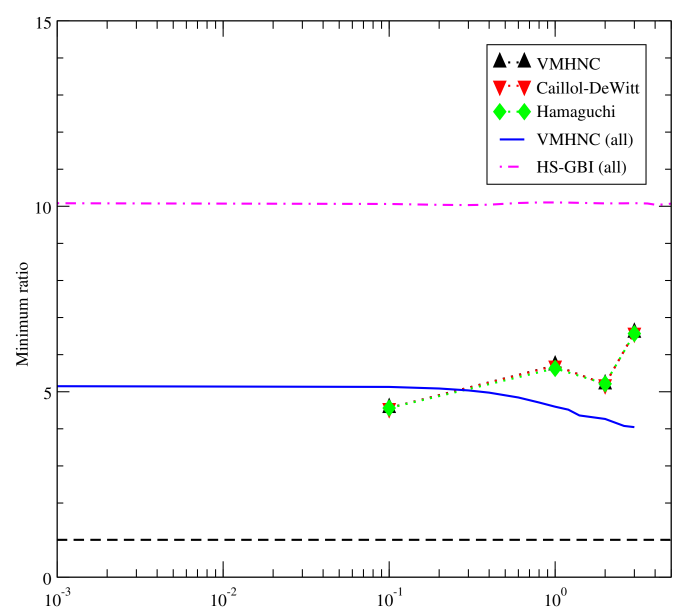

Fig. 1. Minimum ratio as a function of screening parameter κ using the VMHNC [15], the Caillol–DeWitt [13,23], or the Hamaguchi [10,11,15] equation of state with a κ-dependent normalized-viscosity solid-density aluminum [8]. We plot also results obtained using the hard-sphere Gibbs-Bogolyubov inequality (HS-GBI) [14] or the VMHNC with a κ-independent normalized-viscosity fit [8] (all). The dashed-line is the string theory lower-bound equal to one.