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Systematic ranging and late warning asteroid impacts

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ABSTRACT

We describe systematic ranging, an orbit determination technique suitable to assess the near-term Earth impact hazard posed by newly discovered asteroids. For these late warning cases, the time interval covered by the observations is generally short, perhaps a few hours or even less, which leads to severe degeneracies in the orbit estimation process. The systematic ranging approach gets around these degeneracies by performing a raster scan in the poorly-constrained space of topocentric range and range rate, while the plane of sky position and motion are directly tied to the recorded observations. This scan allows us to identify regions corresponding to collision solutions, as well as potential impact times and locations. From the probability distribution of the observation errors, we obtain a probability distribution in the orbital space and then estimate the probability of an Earth impact. We show how this technique is effective for a number of examples, including 2008 TC_3 and 2014 AA, the only two asteroids to date discovered prior to impact.

1. Introduction

When an asteroid is first discovered, the few available observations rarely allow the determination of its orbit. Surveys usually capture 3–5 repeat images of the same area of sky with a typical return time of 15–30 min. Over this time interval, an object in the field of view moves with respect to background stars at a roughly linear rate. Thus, the short arc of available data, also called a tracklet (Kubica et al., 2007), provides an estimate of the angular position and motion of the object in the plane of sky, which is not enough to determine a well-constrained six-parameter orbit (Milani and Knežević, 2005).

Despite the degeneracies in the orbit estimation process, it is important to recognize potentially hazardous objects shortly after discovery. In fact, the only two objects discovered prior to an Earth impact, namely 2008 TC_3 and 2014 AA, were both discovered by Kowalski et al. (2008, 2014) of the Catalina Sky Survey (Larson et al., 1998) only about 20 h before striking the Earth. On one hand, 2008 TC_3 was quickly recognized as potential impactor and so it was extensively observed before the impact. The acquired observations allowed the estimation of the orbit of 2008 TC_3 as well as some physical characterization (Jenniskens et al., 2009; Kozubal et al., 2011; Scheirich, 2010). Using the available data, it was possible to predict the impact that took place above Sudan on 2008 October 7[1] (Jenniskens et al., 2009). On the other hand, 2014 AA was not immediately recognized as a possible impactor and so only seven astrometric observations over about 70 min were obtained before the object fell into the Atlantic Ocean on 2014 January 2[2] (Chesley et al., 2015).

When the standard differential correction procedure (e.g., Farnocchia et al., in press; Milani and Gronchi, 2010) to find a least-squares orbit fails, other methods can be used to assess the orbital probability distribution. In particular, Muinonen and Bowell (1993) show how to put a probability density on the phase space of the orbital elements by employing Bayesian inversion theory. By using a Monte Carlo approach one can sample the orbit phase space and thereby derive the probability distribution from the observation errors corresponding to the sampled orbit.

The available observations directly constrain the position and motion of the asteroid in the sky while the distance between the asteroid and the observer (topocentric range) is poorly constrained. Thus, ranging methods are the preferred Monte Carlo approach when only a short arc of observations is available. Virtanen et al. (2001) describe a method called statistical ranging that allows

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[1] http://neo.jpl.nasa.gov/news/2008tc3.html.
[2] http://www.nasa.gov/jpl/asteroid/first-2014-asteroid-20140102.

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0019-1035/

one to generate orbital samples compatible with the observational data. After choosing two observations, the topocentric ranges at the epochs of the two observations are randomly sampled and a corresponding orbit computed. Oszkiewicz et al. (2009) rely on the statistical ranging technique and use Markov chains to generate an unbiased sequence of orbital samples distributed according to the probability distribution resulting from Bayesian inversion theory.

Chesley (2005) introduces a technique called systematic ranging,[3] which in contrast to Monte Carlo techniques systematically explores a raster in the topocentric range and range-rate space. This technique provides a geometric description of the orbital elements as a function of range and range rate. Moreover, systematic ranging allows one to identify regions of the phase space filled with impact solutions and the corresponding impact times and locations. In this paper we present a detailed description of systematic ranging and show how to derive a probability distribution on the range and range-rate space, which is then mapped to the orbital element space where impact probability estimates can be derived.

2. Systematic ranging

Systematic ranging relies on the fact that a short arc of observations yields a direct estimate of the plane of sky position (right ascension α and declination δ) and motion ($\dot{\alpha}$ and $\dot{\delta}$). These four scalar quantities can be assembled together in the so-called attributable $\mathcal{A} = (\alpha, \delta, \dot{\alpha}, \dot{\delta})$ (Milani and Knežević, 2005) at a chosen time t, which we always set to the time of the first observation. The topocentric range ρ and topocentric range-rate $\dot{\rho}$ are only marginally constrained, if at all. If ρ and $\dot{\rho}$ were known, we would have a full description of the asteroid's topocentric position and velocity in polar coordinates $(\alpha, \delta, \dot{\alpha}, \dot{\delta}, \rho, \dot{\rho})$, which can be easily converted to a Cartesian heliocentric state if the position and velocity of the observer are known. Note that, to account for light-time correction, we correct the epoch of the Cartesian state from that of the attributable by a quantity ρ/c, where c is the speed of light.

To explore the orbit phase space, systematic ranging scans a suitably dense grid in the $(\rho, \dot{\rho})$ space. Such grid needs to be large enough to contain the reasonably possible orbital configurations. In particular, the grid must contain the so-called admissible region (Milani et al., 2004), i.e., the values of $(\rho, \dot{\rho})$ leading to bounded heliocentric orbits. For each grid point we fix the values of $\rho = \rho_i$ and $\dot{\rho} = \dot{\rho}_j$ and find the best fit value of the attributable $\mathcal{A}_{ij}$ that minimizes the cost function:

$$Q = \boldsymbol{v}^T W \boldsymbol{v} \qquad (1)$$

where $\boldsymbol{v}$ is the vector of Observed–Computed astrometric residuals and W is the weight matrix (Farnocchia et al., in press). The minimum of Q is iteratively found via differential corrections:

$$\Delta\mathcal{A} = -\left(B^T W B\right)^{-1} B^T W \boldsymbol{v}, \quad B = \frac{\partial \boldsymbol{v}}{\partial \mathcal{A}}.$$

The starting guess for $\mathcal{A}$ is $(\alpha_1, \delta_1, (\alpha_N - \alpha_1)/(t_N - t_1), (\delta_N - \delta_1)/(t_N - t_1))$, where the subscripts 1 and N refer to the first and the last observation in the tracklet, respectively.

The constrained best-fitting solution can easily be converted to an orbit, which is in turn propagated to find upcoming Earth encounters. Moreover, if the observations contain photometric measurements we also compute the absolute magnitude for each grid point.

Figure 1 shows the application of systematic ranging to 2014 AA by using the seven astrometric observations obtained by the Catalina Sky Survey prior to impact. The orbital elements are shown as a function of $(\rho, \dot{\rho})$. The dashed curve encloses bounded orbits and corresponds to the admissible region of Milani et al. (2004). The dash-dotted line is for grazing impacts: the region on the left of the curve contains impacting solutions.

3. Probability distribution on the range and range rate space

The raster scan presented in the previous section gives a geometric description of how the orbital configuration depends on the value of topocentric range and range rate. The next step is to assign a probability distribution to this space.

As is common practice, given a set of optical observations we assume that the observation errors $\boldsymbol{v}$ are normally distributed according to a weight matrix W, i.e., their probability density f_{err} is:

$$f_{err}(\boldsymbol{v}) \propto \exp(-0.5\ \boldsymbol{v}^T W \boldsymbol{v}).$$

It is typical to use a diagonal weight matrix where the individual weights are $1/\sigma^2$ (σ is the standard deviation of the error in the astrometric positions). Table 1 shows the data weights we currently use for the most productive discovery and follow-up stations. The chosen weights are based on our experience and account for the fact that discovery observations can sometimes be problematic because they are not targeted on the object.

In principle, it is possible to consider the photometric residuals together with the astrometric ones. However, photometric measurements have a much larger uncertainty than that of the astrometry and can be affected by significant biases (Jurić et al., 2002). Moreover, in case of a magnitude trend, it is hard to tell if the varying luminosity is due to a rapidly changing topocentric distance rather than the asteroid's rotation. Therefore, we do not use the information obtained from the photometric residuals.

According to Bayesian inversion theory (Muinonen and Bowell, 1993), the posterior probability density function for $(\rho, \dot{\rho})$ is:

$$f_{post}(\rho, \dot{\rho}) \propto f_{err}(\boldsymbol{v}(\rho, \dot{\rho})) f_{prior}(\rho, \dot{\rho}),$$

where f_{prior} is a prior distribution on the $(\rho, \dot{\rho})$ space.[4] The selection of f_{prior} is to some extent arbitrary and far from trivial (Janyes, 1968), and yet can significantly affect the posterior probability distribution, especially for short observation arcs.

Whatever the choice of f_{prior}, we always add a crude constraint from the population model by setting $f_{prior} = 0$ for hyperbolic orbits, flagging objects where the astrometric errors favor an unbounded orbit.

3.1. Jeffreys' prior

A mathematically sound choice is Jeffreys' prior (Granvik et al., 2009):

$$f_{prior}(\rho, \dot{\rho}) = \sqrt{\det\left(\frac{\partial \boldsymbol{v}}{\partial(\rho, \dot{\rho})}^T\ W\ \frac{\partial \boldsymbol{v}}{\partial(\rho, \dot{\rho})}\right)}.$$

Note that the partials are the total derivatives of $\boldsymbol{v}$ with respect to $(\rho, \dot{\rho})$, which means that they account for the fact that the best-fit attributable $\mathcal{A}$ changes as a function of $(\rho, \dot{\rho})$.

Among other things, Jeffreys' prior secures the invariance of the probability distribution when changing variables. However, since there is more sensitivity of the residuals for small topocentric distances, Jeffreys' prior tends to favor orbital configurations where

[3] This technique was actually introduced by Tholen and Whiteley in 2002, but the paper was never published.

[4] It is often convenient to use a logarithmic scale for ρ to achieve a better resolution at small topocentric distances. In that case the probability density has to be multiplied by ρ.