Icarus 258 (2015) 239–266

Contents lists available at ScienceDirect

Icarus

journal homepage: www.elsevier.com/locate/icarus

Tidal Love numbers of membrane worlds: Europa, Titan, and Co.

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ARTICLE INFO

Article history:
Received 27 March 2015
Revised 27 May 2015
Accepted 5 June 2015
Available online 14 June 2015

Keywords:
Tides, solid body
Planetary dynamics
Europa
Titan

ABSTRACT

Under tidal forcing, icy satellites with subsurface oceans deform as if the surface were a membrane stretched around a fluid layer. 'Membrane worlds' is thus a fitting name for these bodies and membrane theory provides the perfect toolbox to predict tidal effects. I describe here a new membrane approach to tidal perturbations based on the general theory of viscoelastic–gravitational deformations of spherically symmetric bodies. The massive membrane approach leads to explicit formulas for viscoelastic tidal Love numbers which are exact in the limit of zero crust thickness. Formulas for load Love numbers come as a bonus. The accuracy on k_2 and h_2 is better than one percent if the crust thickness is less than five percents of the surface radius, which is probably the case for Europa and Titan. The new approach allows for density differences between crust and ocean and correctly includes crust compressibility. This last feature makes it more accurate than the incompressible propagator matrix method. Membrane formulas factorize shallow and deep interior contributions, the latter affecting Love numbers mainly through density stratification. I show that a screening effect explains why ocean stratification typically increases Love numbers instead of reducing them. For Titan, a thin and dense liquid layer at the bottom of a light ocean can raise k_2 by more than ten percents. The membrane approach can also deal with dynamical tides in a non-rotating body. I show that a dynamical resonance significantly decreases the tilt factor and may thus lead to underestimating Europa's crust thickness. Finally, the dynamical resonance increases tidal deformations and tidal heating in the crust if the ocean thickness is of the order of a few hundred meters.

1. Introduction

Tidal Love numbers are three numbers quantifying the response of a spherically symmetric body to tides or to changes in rotation or orientation. Their computation is required for all applications in which global deformations intervene: tidal or despinning tectonics, tidal heating, true polar wander, tidal currents in Titan's seas (applications discussed in Beuthe (2015) except for the last one, see Tokano et al. (2014)). Conversely, measuring Love numbers helps to constrain interior models.

Membrane worlds refer to planetary bodies with a thin shell floating on a liquid layer (Beuthe, 2015). 'Thin shell' means here that deformations can be predicted with simple membrane equations instead of the more complicated thick shell theory. In practice, membrane theory applies to shells having a thickness less than five to ten percents of the surface radius. The term is thus perfectly suited to the large Galilean and Saturnian icy satellites for which electric, magnetic (including auroral), and gravity data point to the existence of a global ocean close to the surface (Table 1). Though observations are still lacking, Triton and Ceres are candidate membrane worlds (Nimmo and Spencer, 2015; Hand, 2015); many smaller bodies could also enclose an ocean but are unlikely to satisfy the membrane assumption (Hussmann et al., 2006). In this paper, I choose Europa and Titan as case studies because of the available data, their potential for future missions, and their differences in internal structure and orbital period (Table 2).

In a previous paper, I obtained analytical formulas for tidal Love numbers using thin shell theory in the membrane limit (Beuthe, 2015). Although the method was by and large successful (especially regarding depth-dependent crustal rheology), it was lacking in some respects. First, it required that the floating shell be of the same density as the underlying ocean. In the membrane limit (shell of vanishing thickness), this is equivalent to assuming that the membrane is massless. I will thus call this method the *massless membrane approach*. Second, accurate benchmarking of the tilt factor formula revealed a mismatch associated with shell compressibility. Apparently, the classical equations of thin shell theory are not completely satisfactory regarding their dependence on compressibility. For these two reasons, I develop in this paper an alternative membrane formalism, called the *massive membrane approach*, which is based on the viscoelastic–gravitational equations used to predict tidal deformations in thick shell theory. These equations have been extensively validated through their

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http://dx.doi.org/10.1016/j.icarus.2015.06.008
0019-1035/

Table 1
Large icy satellites: some constraints on their crust thickness d (absolute and relative to the surface radius R) from gravity (G), magnetic (M), auroral (A), and electric (E) data.

	d (km)	d/R (%)	Data	Reference
Europa	<170	<11	G	Anderson et al. (1998)
	<200	<13	M	Zimmer et al. (2000)
	<15	<1	M	Hand and Chyba (2007)
Ganymede	150–330	6–13	A	Saur et al. (2015)
Callisto	<300	<12	M	Zimmer et al. (2000)
Titan	55–80	2–3	E	Béghin et al. (2012)

Table 2
Bulk and orbital parameters of Europa and Titan.

Parameter	Symbol	Europa	Titan	Unit
Spin rate[a]	ω	2.048	0.456	10^{-5} s^{-1}
Surface radius[b]	R	1560.8	2574.76	km
GM[a,c]	GM	3202.74	8978.14	km^3 s^{-2}
Moment of inertia[d]	MoI	0.346	0.341	–
Bulk density[e]	ρ_b	3013	1881.5	kg m^{-3}
Surface gravity[e]	g	1.315	1.354	m s^{-2}
Dynamical parameter[f]	q_ω	4.98	0.395	10^{-4}

[a] JPL satellite ephemerides (http://ssd.jpl.nasa.gov/).
[b] Nimmo et al. (2007) for Europa, Mitri et al. (2014) for Titan.
[c] Iess et al. (2010) for Titan.
[d] Anderson et al. (1998) for Europa, Iess et al. (2012) for Titan.
[e] Computed from GM and R ($G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}).
[f] Computed from Eq. (20).

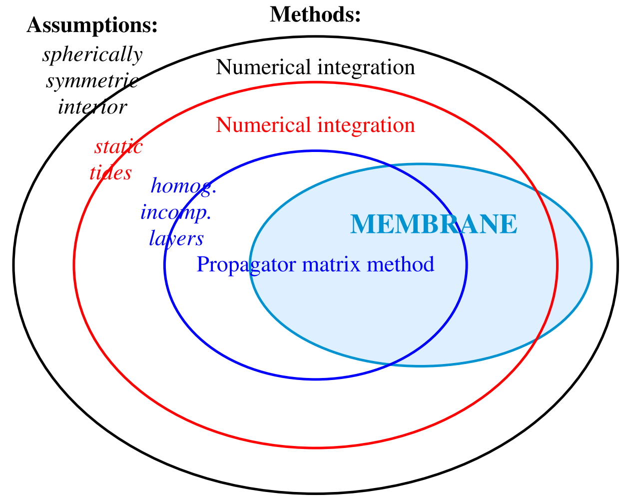

Fig. 1. Domain of validity of the membrane approach in comparison with other methods ('homog. incomp.' = homogeneous and incompressible).

accurate prediction of the frequency spectrum of Earth normal modes (Dahlen and Tromp, 1999).

Though technically complex, the massive membrane approach is based on two simple ideas. The first idea consists in using the vis coelastic–gravitational equations in order to propagate the three unknown Love numbers from the surface to the crust-ocean boundary, where they must satisfy two conditions called *free-slip* and *fluid constraint*. This procedure results in two relations between tidal Love numbers, the $l_n - h_n$ and $k_n - h_n$ *relations*, which depend on the *effective viscoelastic parameters* of the crust. The second idea consists in factorizing the shallow interior from the deep interior: in the static limit of equilibrium tides, the Love numbers of the body with its viscoelastic crust are expressed in terms of the Love numbers of a simpler model (or *fluid-crust model*) in which the crust is fluid-like. Combining these two ideas leads to explicit formulas for Love numbers in terms of crustal parameters and of the deep interior structure. If tides are dynamical, fluid-crust models must be given up but it remains possible to derive membrane formulas for Love numbers in a model with an infinitely rigid mantle. I will show that a dynamical resonance increases surface deformations and tidal heating in the crust as the ocean becomes shallower.

The massive membrane approach is more than 'yet another method' for computing Love numbers. It has the interesting feature that there is an overlap, but no coincidence, between the domains of validity of the membrane approach and of the standard methods (Fig. 1). To be more clear, three successive assumptions are common when computing Love numbers. First, the interior structure of the undeformed body is assumed to be spherically symmetric. Without this assumption, Love numbers do not really make sense although the Love number concept is sometimes extended to flattened bodies in rotation (Wahr, 1981). Numerical integration methods must be used if no other assumption is made (e.g. Tobie et al., 2005). Second, the static limit is often applied because numerical codes tend to diverge at tidal periods if the body contains a fluid layer (tides are particularly slow on Titan). For example, Wahr et al. (2006), Rappaport et al. (2008), and Wahr et al. (2009) use a code assuming the static limit in all layers whereas Mitri et al. (2014) only apply the static assumption to the ocean. Numerical integration remains necessary in the static limit. Third, the interior structure is often discretized as an onion-like superposition of incompressible and homogeneous layers. These rather strong assumptions lead to the *incompressible propagator matrix method* (e.g. Sabadini and Vermeersen, 2004) which provides analytic solutions for two- or three-layer models while models with more layers are easily solved numerically (the propagator matrix method also exists in a dynamical and compressible version which is seldom used for reasons explained in Appendix F). The matrix method is stable when solid layers become fluid-like, contrary to most numerical codes. These qualities make it popular in planetology (e.g. Moore and Schubert, 2000; Hussmann et al., 2002; Roberts and Nimmo, 2008; Jara-Orué and Vermeersen, 2011). By contrast, the membrane approach is based on the thin shell approximation, but it does not require incompressible and homogeneous layers. Compared to the propagator matrix method, the massive membrane approach is simultaneously more restrictive (requiring a thin shell) and more general (allowing for compressibility). As dynamical effects can be included in some cases, one could say the same with respect to codes computing static Love numbers by numerical integration.

2. Viscoelastic–gravitational theory

This section reviews the basics of viscoelastic–gravitational theory that are used in the membrane approach.

2.1. y_i functions and Love numbers

Viscoelastic–gravitational theory describes the deformations of a self-gravitating body with a spherically symmetric internal structure. The deformations can result from tides, rotational flattening, surface loading or free oscillations due to an (Earth) quake. In the standard formalism of Alterman et al. (1959), the equations of motion and Poisson's equation form a set of six differential equations of first order, the solutions of which are six radial functions y_i ($i = 1, \ldots, 6$). Following the conventions of Takeuchi and Saito (1972), y_1 and y_3 are scalars associated with radial and tangential displacements, respectively,

$$(u_r, u_\theta, u_\phi) = \left(y_1, y_3 \frac{\partial}{\partial\theta}, \frac{y_3}{\sin\theta} \frac{\partial}{\partial\phi} \right) U, \tag{1}$$