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Stability of binaries. Part 1: Rigid binaries

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ABSTRACT

We consider the stability of binary asteroids whose members are possibly granular aggregates held together by self-gravity alone. A binary is said to be stable whenever each member is orbitally and structurally stable to both orbital and structural perturbations. To this end, we extend the stability test for rotating granular aggregates introduced by Sharma (Sharma, I. [2012]. J. Fluid Mech., 708, 71–99; Sharma, I. [2013]. Icarus, 223, 367–382; Sharma, I. [2014]. Icarus, 229, 278–294) to the case of binary systems comprised of rubble members. In part I, we specialize to the case of a binary with *rigid* members subjected to full three-dimensional perturbations. Finally, we employ the stability test to critically appraise shape models of four suspected binary systems, viz., 216 Kleopatra, 25143 Itokawa, 624 Hektor and 90 Antiope.

1. Introduction

An increasing number of small objects such as asteroids and trans-Neptunian/Kuiper-belt objects (TNO/KBO), hitherto thought to be solitary, are being identified as binaries; see e.g., Richardson and Walsh (2006), Merline et al. (2002), etc. Currently, about 15% of near-Earth asteroids (NEAs) with sizes greater than 0.3 km are thought to be binary systems, and a similar estimate holds for the main-belt asteroids; see, e.g., Scheirich and Pravec (2009). The fraction of objects thought to be binaries is at least 10% amongst the TNOs, see, e.g., Noll et al. (2008). The members of several of these binary systems are believed to be granular aggregates held together by self-gravity alone. This belief rests primarily upon the low estimated densities of these objects, for example, Itokawa, a possible contact binary, has an estimated density of about 1.9 g cm^{-3}; see Abe et al. (2006). Further support comes from the hypothesis that binaries are often the result of the coming together of particles following catastrophic events such as tidal disruption (Walsh and Richardson, 2006), spin assisted disruption or surface shedding (Walsh et al., 2008), or even impacts (Michel et al., 2001, 2004).

Sharma (2010), henceforth Paper I, considered the equilibrium of rubble binary systems. The binary's members were taken to be tidally-locked, ellipsoidal, granular aggregates orbiting on circular orbits about their common center of mass with their long axes aligned. The equilibrium shapes of fluid binaries has been extensively studied in the context of the classical first- and second-Darwin problems, see, e.g., Chandrasekhar (1969, Ch. 8). While the first Darwin problem dealt with binaries whose secondary was much smaller than the primary, the second Darwin problem addressed binaries with congruent members. Leone et al. (1985) attempted to extend the latter analysis to fluid binaries with unequal members through their so-called "Roche binary approximation"; see also Descamps (2015). Within this approximation, the members orbit at a Keplerian value and the equilibrium shape of each member is found after neglecting the triaxial shape of its companion. The contradictions inherent in their approach were discussed in Paper I, where a self-consistent approach was presented.

Stability analyses of binaries have typically focussed on the orbital stability of these objects, disregarding the response of the binary members as distributed masses that may deform significantly. Extensions to situations wherein the physical extent of the members is retained were pursued by Scheeres (2007), who investigated the rotational fission of contact binaries, and Scheeres (2009), who discussed the dynamical stability of the full[1] planar two-body problem; in both cases the binary consisted of rigid bodies. In astrophysical applications, structural stability of ellipsoidal fluid binaries has, however, been investigated; see Chandrasekhar (1969, Ch. 8). Some recent advances are due to Lai et al. (1993, 1994) who considered the stability of compressible inviscid fluid Roche and Roche-Riemann ellipsoids. These authors

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[1] This indicates that each body's mass distribution is taken into account.

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tested stability by minimizing an energy functional that was allowed to depend on parameters such as the ellipsoid's shape, density, mass, orbital separation, angular momentum, and internal vorticity.

We are interested in the stability of binaries. A binary system will be deemed stable *only* if the orbit *and* the structure of each member are stable to *both* orbital and structural perturbations. Here, by structure we mean the collection of material points that constitute a binary member, and structural stability refers to this collective staying close to its equilibrium configuration. We limit ourselves to affine velocity perturbations of the equilibrium state. This consists of homogeneous structural deformations of a member about its, possibly moving, mass center. Post-perturbation, both the primary and the secondary deform homogeneously. We discuss stability more precisely in Section 4. While the present framework is fairly general, we will present final calculations suitable for binaries with rigid, prolate members.

In the current paper, the energy criterion will be introduced and then specialized to the case of a binary system with rigid members. The stability of granular binaries is investigated in part II of this work. The stability of planar rigid binary system, with near-spherical members, has been investigated previously by Scheeres (2009). There are several reasons to consider the rigid binary system separately. First, a stability analysis of a simple, but important case, will help in clarifying our method. Second, a match between our results in the case of a planar problem with those of Scheeres (2009) will engender confidence both in the final stability prediction and the energy stability criterion. Third, we do not restrict ourselves to the plane, but investigate the full three-dimensional problem. Fourth, rather than expanding the gravitational field of an ellipsoidal object in a series of mass moments about the field of a spherical body, we work with the tidal potentials of ellipsoids, and are so able to observe the effect of shape on the binary's stability. The latter two generalize the stability problem considered by Scheeres (2009). Finally, we will require the stability results for a rigid binary when we consider the stability of granular binaries.

We first derive dynamical equations for a binary system with rigid ellipsoidal members. We generally follow the notation of Paper I, noting changes explicitly. A short primer on tensors is available in Sharma (2009, App. A), and more information may be found in texts such as Knowles (1998).

2. Binary dynamics

We model a binary system as consisting of two ellipsoidal members orbiting about their common center of mass; see Fig. 1. The more massive of the two members is designated as the primary, while the other is the secondary. Both members have associated with them their respective principal axes coordinate systems. The principal coordinate system $\mathcal{P}$ of the primary is defined by the unit vectors $\hat{\mathsf{e}}'_i$, while $\hat{\mathsf{e}}''_i$ identify the principal frame $\mathcal{S}$ of the secondary. All other vectors and tensors related to the primary (secondary) will be labeled by a subscript 'P' ('S'), while associated scalars will be indicated by single primes '$\prime$' (double primes '$\prime\prime$'), e.g., ρ' and ρ'' refer to the densities, respectively, of the primary and the secondary. Similarly, the components of a vector or tensor quantity in $\mathcal{P}$ ($\mathcal{S}$) will be indicated by a single prime (double prime), e.g., the location r_P of the center of mass of the primary with respect to the binary's mass center C may be expressed as $r'_{i_P}\hat{\mathsf{e}}'_i$ in $\mathcal{P}$, or as $r''_{i_P}\hat{\mathsf{e}}''_i$ in $\mathcal{S}$. The distance of the primary from C remains, however, r' in both $\mathcal{P}$ and $\mathcal{S}$. Finally, $\hat{\mathsf{e}}_P$ and $\hat{\mathsf{e}}_S$, with $\hat{\mathsf{e}}_P = -\hat{\mathsf{e}}_S$, are unit vectors that orient the primary and the secondary with respect to each other. The location of the primary with respect to the secondary, and vice versa are, respectively, $\mathsf{R}_P = R\hat{\mathsf{e}}_P$ and $\mathsf{R}_S = R\hat{\mathsf{e}}_S$, with R being the separation between the members of the binary system.

For investigating stability, it is necessary to rewrite governing equations in an appropriate coordinate system $\mathcal{O}$ that rotates at $\omega(t)$ and has its origin at the binary's mass center. We define $\mathcal{O}$ in Section 4.1. Relative to $\mathcal{O}$, the frames $\mathcal{P}$ and $\mathcal{S}$ rotate at, respectively, $\omega_P(t)$ and $\omega_S(t)$. In the sequel, we will associate to any angular velocity $\omega(t)$ an anti-symmetric *angular-velocity tensor* $\boldsymbol{\Omega}(t)$, so that, for every vector x,

$$\omega \times \mathsf{x} = \boldsymbol{\Omega}\cdot\mathsf{x}; \tag{1}$$

ω is the *axial vector* of $\boldsymbol{\Omega}$. Unless otherwise stated, all time derivatives will be with respect to an observer in the rotating frame $\mathcal{O}$.

2.1. Rigid body motion

Consider the binary shown in Fig. 1. Let the binary have rigid members with w_P and w_S being the *angular velocities* of, respectively, the primary and the secondary, as observed in a rotating frame $\mathcal{O}$. Material points within a member are located relative to its mass center by x. Invoking (1), we may write,

$$\dot{\mathsf{x}}_S = \mathsf{w}_S\times\mathsf{x}_S = \boldsymbol{W}_S\cdot\mathsf{x}_S \quad\text{and}\quad \dot{\mathsf{x}}_P = \mathsf{w}_P\times\mathsf{x}_P = \boldsymbol{W}_P\cdot\mathsf{x}_P. \tag{2}$$

Adding the motions of the mass centers, the velocities of material points within the two members become

$$\mathsf{v}_S = \dot{\mathsf{r}}_S + \boldsymbol{W}_S\cdot\mathsf{x}_S \quad\text{and}\quad \mathsf{v}_P = \dot{\mathsf{r}}_P + \boldsymbol{W}_P\cdot\mathsf{x}_P. \tag{3}$$

Paper I derives dynamical equations governing a binary with homogeneously deforming members. These are now written in the rotating frame $\mathcal{O}$, and specialized to the case of a rigid binary by setting the velocity gradient tensors $\boldsymbol{L}_S$ and $\boldsymbol{L}_P$ to be, respectively, $\boldsymbol{W}_S$ and $\boldsymbol{W}_P$. We find:

$$\left(\dot{\boldsymbol{W}}_S + \boldsymbol{W}_S^2\right)\cdot\boldsymbol{I}_S = -\overline{\boldsymbol{\sigma}}_S V'' + \boldsymbol{M}_S^T - \left(\dot{\boldsymbol{\Omega}} + \boldsymbol{\Omega}^2 + 2\boldsymbol{\Omega}\cdot\boldsymbol{W}_S\right)\cdot\boldsymbol{I}_S \tag{4a}$$

and $$\dot{\boldsymbol{I}}_S = \boldsymbol{W}_S\cdot\boldsymbol{I}_S - \boldsymbol{I}_S\cdot\boldsymbol{W}_S, \tag{4b}$$

and

$$\left(\dot{\boldsymbol{W}}_P + \boldsymbol{W}_P^2\right)\cdot\boldsymbol{I}_P = -\overline{\boldsymbol{\sigma}}_P V' + \boldsymbol{M}_P^T - \left(\dot{\boldsymbol{\Omega}} + \boldsymbol{\Omega}^2 + 2\boldsymbol{\Omega}\cdot\boldsymbol{W}_P\right)\cdot\boldsymbol{I}_P \tag{5a}$$

and $$\dot{\boldsymbol{I}}_P = \boldsymbol{W}_P\cdot\boldsymbol{I}_P - \boldsymbol{I}_P\cdot\boldsymbol{W}_P, \tag{5b}$$

where the dot $(\dot{\ })$ indicates time derivative with respect to an observer in $\mathcal{O}$, $\overline{\boldsymbol{\sigma}} = \int_V \boldsymbol{\sigma}\,dV/V$ is the volume-average stress tensor within a rigid binary member,

$$\boldsymbol{I} = \int_V \rho\mathsf{x}\otimes\mathsf{x}\,dV \tag{6a}$$

and $$\boldsymbol{M} = \int_V \rho\mathsf{x}\otimes\mathsf{b}\,dV, \tag{6b}$$

are, respectively, the inertia tensor of a member and the external moment tensor acting on it, and ρ and V are the density and volume of a member, respectively. We recall that the outer product $\mathsf{a}\otimes\mathsf{b}$ of two vectors is a second-order tensor whose components $(\mathsf{a}\otimes\mathsf{b})_{ij} = a_i b_j$.

Both the average stress within the rigid members, and Euler's equation governing their angular velocities, are included in the first of (4) and (5). The average stress may be extracted by taking the symmetric parts of these equations. The anti-symmetric parts of (4a) and (5a) yield, respectively,

$$\boldsymbol{J}_S\cdot\dot{\omega}_S + \omega_S\times(\boldsymbol{J}_S\cdot\omega_S) = \mathsf{T}_S \tag{7a}$$

and $$\boldsymbol{J}_P\cdot\dot{\omega}_P + \omega_P\times(\boldsymbol{J}_P\cdot\omega_P) = \mathsf{T}_P, \tag{7b}$$

where