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Gravity field expansion in ellipsoidal harmonic and polyhedral internal representations applied to Vesta

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ABSTRACT

A 20th degree ellipsoidal harmonic gravity field of Vesta is determined by processing radiometric Doppler and range data from the Dawn mission. The gravity field shows sensitivity up to degree 18 and the coefficients are globally determined on average to degree 15. Gravity anomalies are mapped to the Brillouin ellipsoid (304 × 289 × 247-km), which is a substantially closer fit to the surface than the reference ellipsoid (290 × 290 × 265-km) used to map the conventional spherical harmonic series, especially near the poles. Two models of internal structure are subsequently explored, in which density variations are permitted in the uppermost layer (i.e., crust) in order to explain Vesta's local gravitational signature. These models include the case of a two-layer model with an average crustal thickness of 55.5 km and a three-layer model with an average crustal thickness of 22.4 km. For both two-layer and three-layer scenarios, the Bouguer gravity anomaly is minimized for a crustal density of 2970 kg/m^3. The remaining Bouguer anomalies can be explained by lateral crustal density variation of 2310–3440 kg/m^3 and 2660–3240 kg/m^3 for the 22.4 km and 55.5 km crustal thickness models, respectively. The general trend of the estimated lateral crustal densities for the two cases is very similar, with a wider range for the 22.4 km case due to a thinner crust. This indicates that a thick crust (e.g., 55.5 km) would be more favorable for minimizing the range of lateral crustal density variations. Consideration of independent geochemical and petrological constraints suggests that a three-layer model is a more appropriate representation of Vesta's internal structure, despite the fact that two-layer models provide a satisfactory fit to gravity data alone. In detail, it is found that densities derived from gravity data assuming three-layer models and those derived from the howardite–eucrite–diogenite meteorites and estimates of plausible bulk-Vesta composition show an excellent level of mutual consistency.

1. Introduction

The Dawn spacecraft entered orbit around Vesta on July 16, 2011, and departed Vesta on September 5, 2012. During this period, Deep Space Network (DSN) stations tracked the spacecraft at different altitudes as a part of the Dawn gravity science experiment and acquired X-band (7.179 GHz uplink and 8.435 GHz downlink) two-way coherent Doppler and range data (Konopliv et al., 2011). The mission also mapped Vesta's surface using its onboard framing camera (5.5° field of view). The images were used to create a high-resolution Vesta shape model with spatial resolution ranging from 20 to 260 m (Gaskell, personal communication). Both radiometric and optical measurements were processed to determine a 20th degree and order spherical harmonic gravity field and orientation parameters of Vesta (Konopliv et al., 2014). Detailed harmonic and statistical analyses of the gravity and topography of Vesta are discussed in a companion paper (Bills et al., 2014).

The purpose of this paper is to represent Vesta's gravity field using alternate basis functions and to discuss possible constraints applicable to Vesta's internal structure. The gravitational potential of spherical bodies, such as planets, is usually modeled using the spherical harmonic expansion (Kaula, 1966; Heiskanen and Moritz, 1967), i.e.,

$$U_s(r,\lambda,\phi) = \frac{GM}{r} + \frac{GM}{r}\sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(\frac{R}{r}\right)^n \bar{P}_{nm}(\sin\phi)[\bar{C}_{nm}\cos(m\lambda) + \bar{S}_{nm}\sin(m\lambda)], \quad (1)$$

where G is the gravitational constant, M is the mass of the central body, n is the degree, m is the order, $\bar{P}_{nm}$ are the fully normalized associated Legendre functions, $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are the fully normalized

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spherical harmonic coefficients (the corresponding normalized zonal harmonics are $\bar{J}_n = \bar{C}_n 0$), R is the reference radius of the body (265-km for Vesta), ϕ is the latitude, and λ is the longitude. The acceleration can be computed by taking the gradient of Eq. (1). Given a field point, i.e., a point where the potential is evaluated, the spherical harmonic series generally converge outside of the Brillouin sphere (292.7-km for Vesta) and generally diverges inside the Bjerhammar sphere (212.3-km for Vesta) (Grafarend and Engels, 1994). Note that the Brillouin sphere is the external sphere defined by the largest surface radius and the Bjerhammar is the internal sphere with the smallest surface radius. Fig. 1 shows Brillouin and Bjerhammar spheres of Vesta. Testing for convergence between these two spheres is a non-trivial problem. For practical purposes, spherical harmonic series converge on the 290 × 265-km ellipsoid (Konopliv et al., 2014) shown in Fig. 1, which yields 32.4 km above the surface at the north pole, 40.1 km above the surface at the south pole, and 25.8 km above the surface on average. Note that the average altitude was computed from $(\sum_i a_i v_i)/v_c$, where a_i is the altitude of a field point, v_i is the volume of the differential crustal element at the field point, v_c is the total volume of the crust, and the summation is done over the entire crustal volume.

For geophysical studies, it is often desired to map gravity close to the surface in order to avoid gravity signal attenuation. Considering Vesta's shape being closer to a tri-axial ellipsoid than a sphere, an ellipsoidal harmonic expansion is a natural, and arguably more optimal, basis function for representing gravity (Garmier and Barriot, 2001; Garmier et al., 2002). The gravitational potential using an ellipsoidal harmonic expansion can be represented as (Garmier and Barriot, 2001):

$$U_e(\lambda_1, \lambda_2, \lambda_3) = GM \sum_{n=0}^{\infty} \sum_{m=1}^{2n+1} \bar{\alpha}_{nm} \frac{F_{nm}(\lambda_1)}{F_{nm}(R_e)} \bar{E}_{nm}(\lambda_2) \bar{E}_{nm}(\lambda_3), \tag{2}$$

where $\bar{\alpha}_{nm}$ are the normalized ellipsoidal harmonic coefficients, F_{nm} are the Lamé function of the second kind, and $\bar{E}_{nm}$ are the normalized Lamé function of the first kind. In this study, the computation of Lamé functions is done in quadruple-precision, which is shown to be stable up to about degree 24. The ellipsoidal coordinates λ_1, λ_2, and λ_3 are defined as the three real roots of the conic equation, i.e.,

$$\frac{x^2}{s^2 - a^2} + \frac{y^2}{s^2 - b^2} + \frac{z^2}{s^2 - c^2} = 1, \tag{3}$$

where x, y, and z are the corresponding Cartesian coordinates and a, b, and c are the reference ellipsoid radii. In Eq. (2), λ_1 serves a role similar to the radius ($r = \sqrt{x^2 + y^2 + z^2}$), and λ_2 and λ_3 are essentially equivalent to latitude (ϕ) and longitude (λ). Moreover, the ratio $F_{nm}(\lambda_1)/F_{nm}(R_e)$ plays a role similar to the ratio R/r and the product $\bar{E}_{nm}(\lambda_2)\bar{E}_{nm}(\lambda_3)$ is similar to the product $\bar{P}_{nm}(\sin\phi)\cos(m\lambda)$ or $\bar{P}_{nm}(\sin\phi)\cos(m\lambda)$ in the spherical harmonic expansion (Garmier and Barriot, 2001; Garmier et al., 2002). For each degree, the number of ellipsoidal harmonic coefficients is the same as in the spherical harmonic expansion, i.e., $\bar{\alpha}_{nm}$ are equivalent to $\bar{C}_{nm}$ and $\bar{S}_{nm}$. For a 20th degree field, there are a total of 441 coefficients.

Another way of representing the external gravity is using a polyhedron model (Werner, 1996). The gravitational potential of a constant-density polyhedron can be represented as:

$$U_p(\mathbf{r}) = \frac{1}{2} G\varrho \left(\sum_{e \in \text{edges}} \mathbf{r}_e^T \mathbf{E}_e \mathbf{r}_e \cdot L_e - \sum_{f \in \text{faces}} \mathbf{r}_f^T \mathbf{F}_f \mathbf{r}_f \omega_f \right), \tag{4}$$

where ϱ is the density, $\mathbf{r}_e^T \mathbf{E}_e \mathbf{r}_e \cdot L_e$ is the contribution of an edge, and $\mathbf{r}_f^T \mathbf{F}_f \mathbf{r}_f \cdot \omega_f$ is the contribution of a face (Werner, 1996; Werner and Scheeres, 1997; Park et al., 2010). Note that the spherical harmonic series may diverge if a field point is inside of the Brillouin sphere, but the polyhedral approach is guaranteed to converge if a field

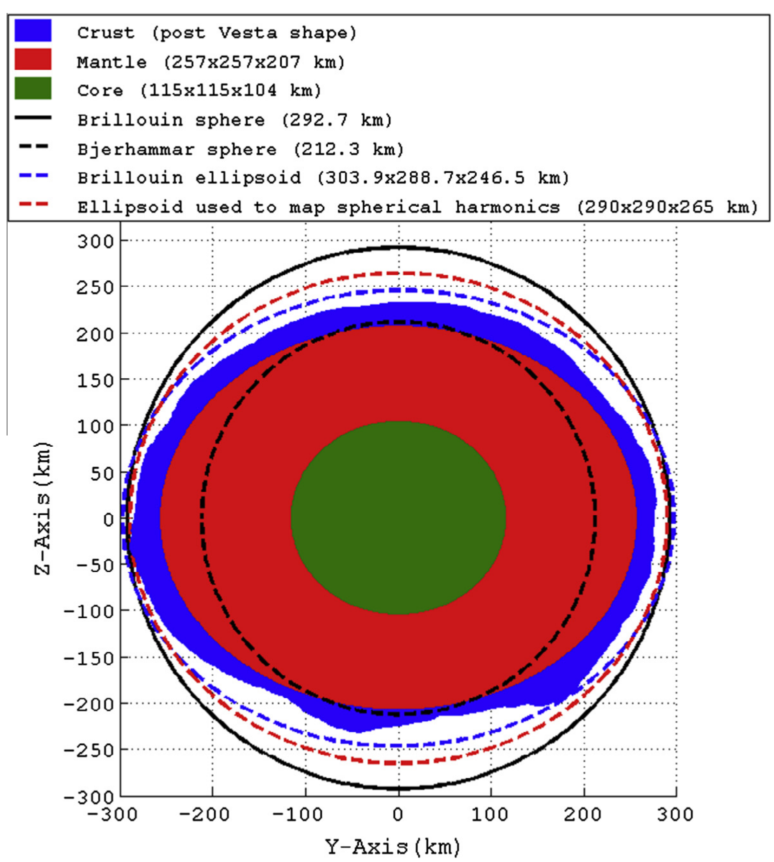

Fig. 1. Various reference spherical/ellipsoidal representations for Vesta and a three-layer model discussed in this paper. Here Vesta is viewed from the east (i.e., x-axis).