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Shape, topography, gravity anomalies and tidal deformation of Titan

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ABSTRACT

Gravity measurements and elevation data from the Cassini mission have been used to create shape, global topography and gravity anomaly models of Titan that enable an improved understanding of its outer ice I shell structure. We provide constraints on the averaged ice shell thickness and its long-wavelength lateral variations, as well as the density of the subsurface ocean using gravity anomalies, the tidal Love number k_2 measurement and long-wavelength topography. We found that Titan's surface topography is consistent with an approximate isostatically compensated ice shell of variable thickness, likely in a thermally conductive or in a subcritical convective state, overlying a relatively dense subsurface ocean.

1. Introduction

Gravity field measurements (Iess et al., 2010, 2012) and shape models (e.g., Zebker et al., 2009a) provide insight into the interior structure of Titan. Iess et al. (2010, 2012) determined the gravity field of Titan with a spherical harmonic expansion to degree three whereas the abundance of altimetry and SAR-Topography data have allowed the estimation of Titan's shape up to degree 7 (Zebker et al., 2009a). Internal structure models of Titan are constrained, though not uniquely, by its moment of inertia, which can be estimated from the gravity field's quadrupole moments neglecting non-hydrostatic components. The estimated normalized moment of inertia of Titan (MoI $\cong 0.3414 \pm 0.0005$) (Iess et al., 2010) is high relative to that of Ganymede, its closest twin in terms of mass and radius. Non-hydrostatic contributions to degree-2 gravity coefficients might produce an overestimate of the moment of inertia, and even if the more probable value of MoI is 0.34, a value of 0.33 must be considered as a lower bound value (Iess et al., 2010; Gao and Stevenson, 2013). To explain the relatively high moment of inertia, it has been suggested that Titan may have either a fully differentiated structure with a deep interior composed of hydrated silicates—for example, mineral antigorite (Castillo-Rogez and Lunine, 2010; Fortes, 2012) or else that the interior is only partially differentiated and at depth is composed of a mixture of ice and rock (Iess et al., 2010; Mitri et al., 2010a). Serpentines as antigorite are hydrous silicates formed on Earth from anhydrous Fe–Mg minerals during hydrothermal alteration of the oceanic lithosphere or from hydration of peridotide above subducting slabs (e.g. Hilairet et al., 2006). These minerals might be formed on Titan by hydrothermal alteration during differentiation as proposed by Castillo-Rogez and Lunine (2010).

The recently measured tidal Love number, k_2, of Titan indicates that the outer ice I shell is decoupled from the deep interior by a global subsurface ocean (Iess et al., 2012). Previous analysis of Titan's ice shell (Nimmo and Bills, 2010) argued that to explain the apparent non-hydrostaticity of Titan's shape, its ice shell thickness must vary as a function of latitude thereby suggesting that the ice shell is in a conductive state (see also Hemingway et al., 2013). Nimmo and Bills (2010) analysis was based on the fluid Love number ($k_f = 1$) derived from the quadrupole moments of the gravity field assuming the hydrostaticity of Titan (Iess et al., 2010) and the shape model of Zebker et al. (2009a).

We produced global topography and gravity anomaly models of Titan that enable an improved understanding of its outer ice I shell structure. We constrain the thermal state and thickness of Titan's ice shell, as well as its lateral variation using as constraints the tidal Love number k_2 (Iess et al., 2012), gravity anomalies and topographic shape models. The shape model used in our analysis of the ice shell is updated from the one presented in Zebker et al. (2009a) based on more recent SAR-Topography (Stiles et al., 2009) and altimetry datasets (Elachi et al., 2004). Then we show that the high measured value of k_2 indicates that the outer ice shell is overlying a relatively dense subsurface ocean.

2. Method

In this section we present the methods used to provide the shape (Section 2.1), the gravity anomalies (Section 2.2) and to model the tidal deformation of the ice I shell (Section 2.3).

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2.1. Shape model

Zebker et al. (2009a) modeled Titan's shape using surface heights derived from Cassini RADAR altimeter (Elachi et al., 2004) and SAR-Topography (Stiles et al., 2009) data products. Both data products are sparsely distributed and collectively cover only a small area of Titan's surface (~1%). Initially, this dataset was further hindered by an uneven latitudinal distribution which incorporated only minimal coverage near the southern pole, prompting Zebker et al. (2009a) to use constrained inversion methods that penalized solutions which deviated from spherical. The current SAR-Topography and Altimetry datasets encompass a greater than 20% increase in areal coverage and, more significantly, a factor of six increase in the coverage poleward of 60°S as compared to what was available to Zebker et al. (2009a). We have used all of these much more comprehensive data sets (acquired through June 2011) to update the best-fit spherical harmonic expansion of Titan's shape. As expected, the new model differs from previous solutions most substantially in the southern hemisphere.

We modeled Titan's shape ($r_S(\theta,\phi)$ using a spherical harmonic expansion of the form:

$$r_s = \sum_{l\geqslant 0}^{n}\sum_{m\geqslant 0}^{l}\left(C^s_{lm}Y^C_{lm} + S^s_{lm}Y^S_{lm}\right) \tag{1}$$

where C^s_{lm} and S^s_{lm} are the harmonic coefficients of the shape, expressed in meters, and Y^C_{lm} and Y^S_{lm} are real orthonormal spherical harmonic basis functions of degree l and order m:

$$Y^C_{lm} = (-1)^m P_{lm}(\cos\theta)\cos(m\phi) \tag{2}$$

$$Y^S_{lm} = (-1)^m P_{lm}(\cos\theta)\sin(m\phi) \tag{3}$$

For $l = 0$, $r_s = R_T$ the mean radius; θ is the co-latitude, ϕ is the longitude (measured to be positive eastward), and P_{lm} are the un-normalized associated Legendre polynomials (Abramowitz and Stegun, 1965). Best-fit coefficients (and their associated uncertainties) were determined from the unconstrained least-squares solution of Eq. (1) in the presence of known covariance. The data's covariance matrix, C^H, incorporates both the random and systematic errors associated with the Altimetry and SAR-Topography datasets. Accordingly, each element c_{ij} of C^H is defined as:

$$c_{ij} = e_i^2 + \sigma_i\sigma_j p_{ij}\delta_{ij} \tag{4}$$

where e_i is the random, or instantaneous, component of the error associated with pointing and algorithmic uncertainty, $\sigma_{i,j}$ is the correlated component of the error associated with errors in spacecraft ephemeris, p_{ij} is a Pearson correlation coefficient between points i and j, and δ_{ij} is a Kronecker delta function that is unity if points i and j are part of the same Titan observation and zero otherwise. See Zebker et al. (2009b) for an explanation of altimetry errors and Stiles et al. (2009) for an explanation of errors associated with the derivation of SAR-Topography points. The Pearson correlation coefficient (p_{ij}) is assumed to exponentially decrease with increasing distance between two points in the same flyby:

$$p_{ij} = \exp\left(-\frac{d_{ij}}{L_c}\right) \tag{5}$$

where d_{ij} is the distance between points i and j on Titan's surface and L_c is an assumed e-folding distance (L_c = 1000 km). The value of L_c was chosen to represent the approximate correlation length of spacecraft ephemeris errors. The solution is only weakly dependent on this value (see Supplementary online material). In order to reduce computational complexity and restrict C^H to practical dimensions, the SAR-Topography and Altimetry data points (along with their associated uncertainties) were binned according to a 2 pixel-per-degree sinusoidal projection prior to solving for the best-fit spherical harmonic coefficients. The variance of the best-fit shape (E^H) is given by:

$$E^H(\theta,\phi) = A(\theta,\phi)\sum^H A(\theta,\phi)^T \tag{6}$$

where $\sum^H$ is the covariance matrix of the best-fit coefficients C_{lm} and S_{lm} ($\sum^H = (\psi^T(C^H)^{-1}\psi)^{-1}$, where ψ is a concatenated matrix of Y^C_{lm} and Y^S_{lm} evaluated at the known topography points) and A is a concatenated matrix of Y^C_{lm} and Y^S_{lm} evaluated at (θ, φ).

We also determined the principal axes of Titan's shape as:

$$a = R_T - \frac{1}{2}C^S_{20} + 3C^S_{22} \tag{7}$$

$$b = R_T - \frac{1}{2}C^S_{20} - 3C^S_{22} \tag{8}$$

$$c = R_T + C^S_{20} \tag{9}$$

The principal axes were also determined by directly solving for the best-fit tri-axial ellipsoid.

As the primary purpose of this effort was comparison with gravity, the shape was only determined through order 6. For these low orders, the solution did not have to incorporate *a priori* constraints, such as penalizing deviations from spherical solutions. When fitting to higher orders, however, the coefficients for large degrees were unstable and needed to include constraints (see Supplementary online material). An independent analysis of Titan's shape by Zebker et al. (2012) incorporates such constraints. For orders six and below, we found that our results are equivalent, to within one standard deviation, to the results of Zebker et al. (2012).

2.2. Topography and gravity anomalies

2.2.1. Topography

We determined the topography of Titan determined as the shape elevation (Section 2.1) referred to the gravity ellipsoid. The gravity ellipsoid, with radius r_{ell}, is defined by the quadrupole equipotential surface given by the quadrupole gravitational coefficients C_{20} ($J_2 = -C_{20}$) and C_{22} (Iess et al., 2012). The topography (h_S) is given by the difference between the elevation of the shape (r_S) given by Eq. (1) and the radius of the reference ellipsoid (r_{ell}):

$$h_S = r_S - r_{ell} \tag{10}$$

The gravitational potential referred to the ellipsoid is given by:

$$U_{ell} = -\frac{GM_T}{r}\left[1 + \left(\frac{R_T}{r}\right)^2 (C_{20}P_{20} + C_{22}\cos 2\phi)\right] + \left[-\frac{1}{3}\omega^2 r^2(1 - P_{20})\right] + \left[\frac{1}{2}\frac{GM_S}{a_T^3}r^2\left(P_{20} - \frac{1}{2}P_{22}\cos 2\phi\right)\right] \tag{11}$$

where G is the gravitational constant, M_T and M_S are the mass of Titan and Saturn, respectively, r is the radial distance, R_T is the mean radius of Titan, ϕ is the longitude, ω is the spin rate, a_T the semi-major axis of Titan's orbit, and P_{20} and P_{22} are associated Legendre functions. The last two terms in the expression of Eq. (11) give the rotational and tidal contribution to the gravitational potential U_{ell} (see Heiskanen and Moritz, 1975). The spin rate ω is inferred from IAU model (Seidelmann et al., 2006), considering the synchronous rotation of Titan (Meriggiola et al., 2013, in preparation). The used physical parameters are summarized in Table 1.

Finally, the radius of the reference ellipsoid r_{ell} is inferred from Eq. (11) computing the correspondent equipotential surface:

$$r_{ell} = R_T\left[1 + \left(C_{20} - \frac{5}{6}q_r\right)P_{20} + \left(C_{22} + \frac{1}{4}q_r\right)P_{22}\cos 2\phi\right] \tag{12}$$

where q_r and q_t are respectively the contributions due to the rotational and the tidal deformations:

$$q_r = \frac{\omega^2 R_T}{GM_T} = 3.9528 \cdot 10^{-5} \tag{13}$$

$$q_t = -3\left(\frac{R_T}{a_T}\right)^3 \frac{GM_S}{GM_T} = -1.1858 \cdot 10^{-4} \tag{14}$$

where for a synchronous rotation body $q_t/q_r = -3$.

2.2.2. Gravity anomalies

We determined the free air gravity anomalies and Bouguer gravity anomalies for Titan. The free air gravity anomalies are given by:

$$g_{FA} = \frac{GM_T}{r_S^2}\left[\sum_{l\geqslant 3}(l+1)\left(\frac{R_T}{r_S}\right)^l \sum_{m=1}^{l}\left(C_{lm}Y^C_{lm} + S_{lm}Y^S_{lm}\right)\right] \tag{15}$$

where $Y^{C,S}_{lm}$ are the spherical harmonic basis functions. Eq. (15) does not take into account the free air correction for the computation of the free air anomalies as Titan's gravity field was measured from the spacecraft rather than from ground measurements. We determined the free air anomalies using the degree-three of the gravity

Table 1
Physical parameters.

Parameter	Variable	Value	Reference
Gravitational parameter of Titan	GM_T	8978.1394 $km^3\ s^2$	Iess et al. (2010)
Spin rate	ω	22.5769768 deg day^{-1}	Meriggiola et al. (2013, preparation)
Ice density	ρ_{ice}	920–935 kg m^{-3}	e.g. Mitri et al. (2010b)
Mean orbital distance of Titan from Saturn	a_T	$1.2218 \cdot 10^6$ km	e.g. Sotin et al. (2010)
Gravitational parameter of Saturn	GM_S	37931207.7 $km^3\ s^{-2}$	Jacobson et al. (2006)