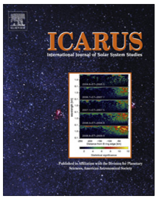
Icarus 225 (2013) 548–557

Contents lists available at SciVerse ScienceDirect

Icarus

journal homepage: www.elsevier.com/locate/icarus



# Saturn layered structure and homogeneous evolution models with different EOSs

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ARTICLE INFO

*Article history:*
Received 24 October 2012
Revised 9 April 2013
Accepted 15 April 2013
Available online 27 April 2013

*Keywords:*
Saturn
Saturn, Interior
Saturn, Atmosphere

ABSTRACT

The core mass of Saturn is commonly assumed to be $10–25M_{\oplus}$ as predicted by interior models with various equations of state (EOSs) and the *Voyager* gravity data, and hence larger than that of Jupiter ($0–10M_{\oplus}$). We here re-analyze Saturn's internal structure and evolution by using more recent gravity data from the *Cassini* mission and different physical equations of state: the ab initio LM-REOS which is rather soft in Saturn's outer regions but stiff at high pressures, the standard Sesame-EOS which shows the opposite behavior, and the commonly used SCvH-i EOS. For all three EOS we find similar core mass ranges, i.e. of $0–20M_{\oplus}$ for SCvH-i and Sesame EOS and of $0–17M_{\oplus}$ for LM-REOS. Assuming an atmospheric helium mass abundance of 18%, we find maximum atmospheric metallicities, $Z_{\rm atm}$ of $7\times$ solar for SCvH-i and Sesame-based models and a total mass of heavy elements, $M_Z$ of $25–30M_{\oplus}$. Some models are Jupiter-like. With LM-REOS, we find $M_Z = 16–20M_{\oplus}$, less than for Jupiter, and $Z_{\rm atm} \lesssim 3\times$ solar. For Saturn, we compute moment of inertia values $\lambda = 0.2355(5)$. Furthermore, we confirm that homogeneous evolution leads to cooling times of only ~2.5 Gyr, independent on the applied EOS. Our results demonstrate the need for accurately measured atmospheric helium and oxygen abundances, and of the moment of inertia for a better understanding of Saturn's structure and evolution.

## 1. Introduction

Saturn is the planet with the lowest mean density in the Solar System. Since the mechanisms that can inflate exoplanets with observed overlarge radii do not hold for the outer planet Saturn, one might thus intuitively think of Saturn as having a smaller core and smaller overall metallicity than Jupiter. However, quantitative estimates on the core mass and on the total heavy element enrichment solely come from interior model calculations, and the same modeling approach applied to both planets just predicts the opposite: an about two times larger maximum core mass and heavy element enrichment for Saturn (Saumon and Guillot, 2004; Guillot and Gautier, 2007). A higher envelope metallicity of Saturn is also supported by the measured atmospheric C:H ratios, which is ~9× solar for Saturn (Fletcher et al., 2009; scaled to the Solar System abundance data of Lodders (2003)) but only 3–5× solar for Jupiter (Atreya et al., 2003).

Certainty about the present core mass and envelope metallicity is desirable because these parameters contain information—albeit not necessarily uniquely (Helled et al., 2010; Boley et al., 2011)—on the formation environment, i.e. on the protosolar disk, and on the process of formation.

Models by Saumon and Guillot (2004), hereafter SG04, are often considered the standard of what we know today about Saturn's present internal structure in terms of core mass and heavy element enrichment (e.g., Alibert et al., 2005; Dodson-Robinson et al., 2010), for mainly two reasons. First, these models have been computed for various physical equations of state (EOS) for Saturn's likely main constituents H and He that also give acceptable solution for Jupiter's interior and evolution (the EOSs SCvH-i, LM-H4, LM-SOCP). Independent on the EOS, the possible core mass range was found to be ~$10–25M_{\oplus}$, while for Jupiter ~$0–10M_{\oplus}$. Second, a wide range of input parameters was accounted for such as a the position of an internal layer boundary that separates a helium-poor, outer from a helium-rich, inner envelope. However, SG04 computed constant metallicity envelope models only, an assumption that tremendously restricts the resulting range of interior models.

In earlier models by Gudkova and Zharkov (1999) and Guillot (1999), the metallicity was allowed to vary across the internal layer boundary. As a consequence, zero-core mass models with high heavy element enrichment in the deep envelope were found for both Jupiter and Saturn.

The new *Cassini* gravity data with their tight observational error bars, and also long-term observational data of the saturnian system (Jacobson et al., 2006; Anderson and Schubert, 2007) raised hope to better constrain Saturn's internal structure. Surprisingly, the most recent Saturn models based on those gravity data cover an even bigger, minimum core mass range of ~$0–30M_{\oplus}$ (Anderson

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0019-1035/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.icarus.2013.04.018

and Schubert, 2007; Helled et al., 2009a; Helled, 2011). Therefore, Helled (2011) suggests to measure the axial moment of inertia as an additional constraint. Her models, however, employ *empirical* pressure–density relations that may reach out of the realm of physical EOS which agree with the available experimental data (see, e.g. SG04; Holst et al., 2012).

Our Saturn models are the first that are based on both physical equations of state and the *Cassini* data. Not is it the purpose of this work to better constrain the core mass: this cannot be achieved within the standard three-layer modeling approach, which is adopted in this work. Instead, we here investigate the overall behavior of core mass, atmospheric metallicity, and deep envelope metallicity on the input parameters: we vary the position of an internal layer boundary in order to recall its influence on the core mass, see also Guillot and Gautier (2007); we exchange the EOS of the envelope material (LM-REOS, SCvH-i EOS, Sesame EOS), and we adopt two different periods of rotation of 10 h 32 m and 10 h 39 m. In lack of accurate observations, we make predictions on the possible helium and heavy element mass fractions in Saturn's atmosphere in dependence on the $J_4$ value and the uncertainty in the rotational period. Our results on the atmospheric helium abundance can serve as constraints for future models of He-sedimentation in Saturn, as long as Saturn's atmospheric He:$H_2$ ratio is not accurately measured.

Observations of young stellar systems and protostellar disks commonly point to formation of the giant planets within a few Myr (Strom et al., 1993), implying a billions-of-years-old planet should have the same age as its host star. However, homogeneous evolution calculations for Saturn, which are mainly based on the SCvH-i EOS, generally yield cooling times of 2–3 Gyr (Saumon et al., 1992; Fortney et al., 2011), about only *half* of the age of the Sun. This implies a higher luminosity of present Saturn than it should have if the underlying assumption of homogeneous evolution would hold. Despite the obvious failure of this assumption, we here adopt it once more in order to investigate the influence of the EOS on the cooling time.

In Section 2.1 we describe our modeling procedure. Section 2.2 is devoted to a detailed description of the observational data, and Section 2.4 to the applied EOSs. Our results are presented in Section 3. In Section 3.1 we investigate the influence on different H–He–EOS on Saturn's structure and in Section 3.2 of the atmospheric He abundance and rotation rate. In Section 3.3 we give the values for the non-dimensional moment of inertia. Section 3.4 contains the cooling curves. Section 4 includes a discussion on the implications for Saturn's formation process (4.3), on the applicability of the three-layer assumption in the presence of He rain (4.4), and a summary of our main findings (4.6).

## 2. Methods

### 2.1. Planetary structure modeling

For understanding the interior of giant gas planets like Saturn it is necessary to consider the gravitational field of the planet. The shape of the field is influenced by different effects. Saturn for instance has primarily the form of an ellipsoid due to its rapid rotation, which can be seen from the rather high ratio of centrifugal to gravitational forces, $q = \omega^2 R_{\mathrm{eq}}^3/(GM)$, where $\omega$ is the angular velocity, $R_{\mathrm{eq}}$ is the equatorial radius, and $M$ the total mass. For Saturn, $q \sim 0.155$ with an uncertainty of 0.004 due to the uncertainty in the rotation period and equatorial radius (see Section 2.2), for Jupiter, $q = 0.089$, and for the Sun, $q = 0.00002$. Tidal forces caused by the gravity of the moons or the parent star can also change the form of a planet's gravity field. While this effect can be important for close-in exoplanets it is tiny for Saturn and has not been measured yet for any giant planet in the Solar System. To assess the rotationally induced deformation, the gravity field $\Phi^{(e)}$ exterior to the mass $M$ is expanded into a series of Legendre polynomials $P_{2n}$, where the expansion coefficients $J_{2n}$ are the gravitational moments at the equatorial reference radius $R_{\mathrm{eq}}$,

$$J_{2n} = -\frac{1}{MR_{\mathrm{eq}}^{2n}} \int d^3r \rho(r,\theta) r^{2n} P_{2n}(t). \tag{1}$$

Being integrals of the internal mass distribution over the volume enclosed within the geoid of equatorial radius $R_{\mathrm{eq}}$, the $J_{2n}$ can be written as depth-dependent functions $J_{2n}(l)$ whose values increase continuously from the center outward until the observed values $J_{2n}^{(\mathrm{obs})}$ are reached at the geoid's mean radius $l = R_{\mathrm{m}}$. As a measure for the contribution $dJ_{2n}$ of a shell at $l$ and extension $dl$ to $J_{2n}^{(\mathrm{obs})}$ we can define the normalized contribution function

$$c_{2n}(l) = \frac{(dJ_{2n}/dl)|_l}{\int dl'(dJ_{2n}/dl')}. \tag{2}$$

For modeling Saturn we use the same method and code as in Nettelmann et al. (2012) for Jupiter. We adopt the standard three-layer structure with two envelopes and a core. The composition of each of the envelopes is diverted into the three components hydrogen, helium, and heavy elements, whereas the core consists of heavy elements only. The helium mass fractions and the metallicities (i.e. the heavy element mass fractions) are parameterized by $Y_1$,$Z_1$ and $Y_2$,$Z_2$ for the outer and the inner envelope, respectively. This implies the assumption of homogeneous envelopes. The transition between them occurs at the transition pressure $P_{1-2}$ which is a free parameter. As observational constraints we take into account $R_{\mathrm{eq}}$, $\omega$, the total mass $M_{\mathrm{Sat}}$, the temperature $T_1$ at the 1 bar level of the planet, and the lowest order moments $J_2$ and $J_4$.

For given values of $Y_1$ and of the mean helium abundance $Y$, $Y_2$ is adjusted to fit $Y$, while $Z_1$ and $Z_2$ are adjusted to fit $J_2$ and $J_4$. Mass conservation is then ensured by the choice of the core mass $M_{\mathrm{core}} = m(R_{\mathrm{core}})$.

### 2.2. Observational constraints

While the *Cassini* mission could provide tight constraints on Saturn's gravity field, there are still important remaining uncertainties, in particular in Saturn's period of rotation, equatorial radius, and the atmospheric helium abundance.

#### 2.2.1. Period of rotation

Prior to the *Cassini* observations, Saturn's period of rotation was taken to be 10 h 39 m 24s, the detected periodicity in the kilometric radio emissions of Saturn's magnetic field as measured by the Voyager I and II spacecraft (Desch and Kaiser, 1981). *Cassini* however revealed a prolongation of this period by several minutes within just 20 years; thus the observed magnetic field modulations may not reflect the rotation of Saturn's deep interior (Gurnett et al., 2007). On the other hand, while alternative methods of deriving the rotation rate from observed wind speeds make assumptions that may not hold true, such as the minimum energy of the zonal winds or a minimum height of isobar-surfaces relative to computed geoid surfaces (Anderson and Schubert, 2007; Helled et al., 2009b), that alternative methods just suggest similar values of ~10 h 32 m. We therefore use these values as the uncertainty in Saturn's real solid body rotation period and compute interior models for both periods, i.e. for 10 h 32 m and 10 h 39 m. Note that we neglect here the uncertainty to Saturn's structure from the possibility of differential rotation on cylinders. On the other hand, all observational wind data can well be reproduced by the assumption of solid-body rotation (Helled et al., 2009b) and the effect of zonal