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Modeling the satellite particle population in the planetary exospheres: Application to Earth, Titan and Mars

A. Beth [a,b,1], P. Garnier [a,b,*], D. Toublanc [a,b], I. Dandouras [a,b], C. Mazelle [a,b], A. Kotova [b,c]

[a] Université de Toulouse, UPS-OMP, IRAP, Toulouse, France
[b] CNRS, IRAP, 9 Av. colonel Roche, BP 44346, F-31028 Toulouse cedex 4, France
[c] Max Planck Institute for Solar System Research, Katlenburg-Lindau, Germany

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ABSTRACT

The planetary exospheres are poorly known in their outer parts, since the low neutral densities are difficult to measure in situ. The exospheric models are thus often the main source of information at such high altitudes. We revisit here the importance of a specific exospheric population, i.e. the satellite particles, which is usually neglected in the models. These particles are indeed produced through rare collisions in the exospheres, and may either be negligible or dominate the exospheres of all planets with dense atmospheres in our Solar System, depending on the balance between their sources and losses. Richter et al. (Richter, E., Fahr, H.J., Nass, H.U. [1979]. Planet. Space Sci. 27, 1163–1173) were the first to propose, beyond the Chamberlain's (Chamberlain, J.W. [1963]. Planet. Space Sci. 11, 901–901) rough approximation, a rigorous approach for these particles by using the Boltzmann equation in the Earth exosphere below 3000 km altitude. They pointed out their negligible presence at low altitudes without doing this calculation at higher altitudes. We further investigate this approach at Earth and apply it another planetary exospheres – Mars and Titan – thanks to improvements in the computing power and the collected planetary data. We determine the contribution of the satellite particles densities of light elements (H_2 at Titan, H at Earth and Mars), and show in particular that the H satellite particles may contribute very significantly to the martian exospheric densities at high altitudes. The H_2 satellite particles are also non-negligible at Titan whereas the H satellite population represents only a small fraction of the total density at Earth. Considering collisionless exospheric profiles – such as the Chamberlain (Chamberlain, J.W. [1963]. Planet. Space Sci. 11, 901–901) approach including the ballistic and escaping populations only – could thus lead to significant underestimations of the total densities at high altitudes in some conditions.

1. Introduction

The exosphere is the upper layer of any planetary atmosphere: it is a quasi-collisionless medium where the particle trajectories are more dominated by gravity than by collisions. Above the exobase, the lower limit of the exosphere, the Knudsen number (Ferziger and Kaper, 1972) becomes large, collisions become scarce, the distribution function cannot be considered as Maxwellian anymore and, gradually, the trajectories of the particles are essentially determined by the gravitation and radiation pressure by the Sun. The trajectories of particles, subject to the gravitational force, are completely solved with the equations of motion, which is not the case with the radiation pressure (Bishop and Chamberlain, 1989).

To describe correctly the exospheric population, we distinguish three types of particles: escaping, ballistic and satellite (Chamberlain, 1963; Banks and Kockarts, 1973) (see Fig. 1).

- The escaping particles come from the exobase and have a positive mechanical energy: they can escape from the gravitational influence of the planet with a velocity larger than the escape velocity. These particles are responsible for the Jeans' escape (Jeans, 1921).
- The ballistic particles also come from the exobase but with a negative mechanical energy, they are gravitationally bound to the planet. They reach a maximum altitude and fall down on the exobase if they do not undergo collisions.
- The satellite particles never cross the exobase. They also have a negative mechanical energy but their periapsis is above the exobase: they orbit along an entire ellipse around the planet without crossing the exobase. The satellite particles result from ballistic particles undergoing few collisions mainly near the

* Corresponding author Address: IRAP, 9 avenue du Colonel Roche, BP 44346, F-31028 Toulouse Cedex 4, France.

E-mail addresses: abeth@irap.omp.eu (A. Beth), pgarnier@irap.omp.eu (P. Garnier), dtoublanc@irap.omp.eu (D. Toublanc).

[1] Principal corresponding author.

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exobase. Thus, they do not exist in a collisionless model of the exosphere.

Chamberlain (1963) proposed an approach to estimate the density of each population via Liouville's theorem which states that the distribution function remains constant along a dynamical trajectory. A Maxwellian distribution function is assumed at the exobase and propagated to the upper layers via Liouville's theorem. The density for each population is then derived as the product between the barometric law and a partition function ζ (see Appendix A.1)

$$n(r) = n_{bar}\zeta(\lambda) = n(r_{exo})e^{\lambda-\lambda_{exo}}(\zeta_{bal} + \zeta_{esc}) \tag{1}$$

where λ is the ratio between the gravitational and thermal energies.

$$\lambda(r) = \frac{GMm}{k_B T_{exo} r} = \frac{v_{esc}(r)^2}{U^2} \tag{2}$$

with r the distance from the center of the body, $v_{esc}(r)$ the escaping velocity, U the most probable velocity for the Maxwellian distribution, G the gravitational constant, M the mass of the planet or the satellite and T_{exo} the temperature at the exobase considered constant in the exosphere.

One problem remains: the satellite population cannot be properly estimated via Liouville's equation. The last limit where the distribution function is known is indeed the exobase. However, the satellite particles do not originate directly from the exobase. The approximation for ζ_{sat} (see Section A.1) given by Chamberlain (1963), based on the Liouville theorem, cannot be rigorous and most probably overestimates the importance of the satellite particles. Notably, this approach predicts that the satellite particle density dominates the ballistic and escaping densities at high altitudes for several planets (see Appendix A.2 and Table 4). The only way to estimate them rigorously is to solve their distribution function f_s thanks to the Boltzmann equation, which includes the rare collisions existing in the exosphere. Most of the exospheric models actually consider the exosphere as collisionless and are thus based on the Chamberlain formalism including the ballistic and escaping populations only, whereas some models include arbitrarily the satellite particles by including the ζ_{sat} contribution.

Richter et al. (1979) used the Boltzmann equation to estimate the satellite particles density in the Earth exosphere taking into account the production and loss processes. They determined the satellite population density in the first 2500 km above the exobase

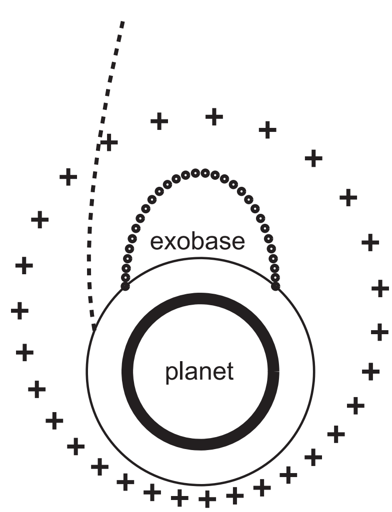

Fig. 1. Three types of trajectories for the exospheric particles: escaping (dotted line), ballistic (o) and satellite particles (+).

and concluded that the satellite particles do not dominate over the ballistic and escaping populations between 500 km and 2500 km. However, they might at higher altitudes since, on Earth, the ratio between the satellite component and the total density increases with altitude.

In this paper, we propose to further investigate this approach and apply it to other planetary exospheres (for Titan and Mars) to estimate rigorously their satellite populations, based on recent planetary data or models. The Section 2 will describe the model, whereas the Section 3 will show the results on Earth with the same conditions as Richter et al. (1979) in order to validate our code. The Sections 4 and 5 will then show the respective results on Titan and Mars. The limits and assumptions of the model will be discussed in Section 6, before a comparative discussion in Section 7 and a conclusion on our study in the last section.

2. Description of the model

2.1. The theoretical approach

Both the ballistic and satellite particles describe an ellipse but the periapsis r_p of the ballistic particles is below the critical level r_c (exobase distance). The presence of satellite particles results from the few collisions undergone by ballistic particles near the exobase thus transforming their trajectories. The distribution function of the satellite particles may be solved via the Boltzmann equation:

$$\frac{Df_s(\vec{r}_s, \vec{v}_s)}{Dt} = P^+(\vec{r}, \vec{v}) + P^-(\vec{r}, \vec{v}) \tag{3}$$

where P^+ and P^- are the local production and loss rates per unit of phase space volume at the position vector $\vec{r}$ with the velocity vector $\vec{v}$.

Along an elliptic path, the Eq. (3) becomes:

$$\frac{Df_s}{Dt} \equiv \frac{\partial f_s}{\partial t} + \vec{v}\cdot\vec{\nabla} f_s = \frac{\partial f_s}{\partial t} + v_s\frac{\partial f_s}{\partial s} = P^+ + P^- \tag{4}$$

with s the curvilinear abscissa on the ellipse and v_s the associated velocity. The local loss rate is directly proportional to the distribution function itself:

$$P^- = -f_s \cdot L_s \tag{5}$$

where L_s is the net loss rate per satellite particle. Finally, we obtain, in the stationary case ($\partial/\partial t = 0$):

$$v_s\frac{df_s}{ds} = P^+ - f_s.L_s \tag{6}$$

The integro-differential Boltzmann equation is reduced to a simple differential equation. To solve it, we suppose that the production rate of satellite particles depends only on the ballistic particles and not on other satellite particles suffering collisions, the satellite population being negligible in the first kilometers above the exobase. This gives:

$$f_s(\vec{r},\vec{v}) = \exp\left(-\int_{s_0}^{s}\frac{L_s'}{v_s'}ds'\right)\left[f_s(\vec{r}_0,\vec{v}_0) + \int_{s_0}^{s}P_s^{+'}\exp\left(\int_{s_0}^{s'}\frac{L_s''}{v_s''}ds''\right)\frac{ds'}{v_s'}\right] \tag{7}$$

where $f_s(\vec{r}_0,\vec{v}_0)$ is the satellite distribution function at the curvilinear abscissa s_0 and $f_s(\vec{r},\vec{v})$ the satellite distribution function at the curvilinear abscissa s. After one revolution around the planet, the satellite particles return to their original position $(\vec{r}_0, \vec{v}_{s0})$.

$$f_s(\vec{r}_0,\vec{v}_0) = \exp\left(-\oint\frac{L_s'}{v_s'}ds'\right)\left[f_s(\vec{r}_0,\vec{v}_0) + \oint P_s^{+'}\exp\left(\int_{s_0}^{s'}\frac{L_s''}{v_s''}ds''\right)\frac{ds'}{v_s'}\right] \tag{8}$$