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The role of tidal torques on the evolution of the system of Saturn's co-orbital satellites Janus and Epimetheus

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ABSTRACT

This paper discusses the effect of tides raised by Saturn on its co-orbital satellites Janus and Epimetheus, and its consequences on the long-term evolution of the co-orbital horseshoe pattern of those moons. This tidal effect is found to produce a loosening of the co-orbital lock of Janus and Epimetheus. The increase of the difference D between their semi-major axes under this process is estimated as 2.77 km/Myear, regardless of whether the moons are composed of monolithic or fractured ice. On the other hand, assuming that the outer edge of Saturn's A ring is permanently maintained by Janus and Epimetheus at their 7:6 resonance, Lissauer et al. [Lissauer, J.J., Goldreich, P., Tremaine S., 1985. Icarus 64, 425–434] have shown that the torques exerted on Janus and Epimetheus due to resonances with Saturn's rings would produce a tightening of the co-orbital lock. The rate of decrease of orbital difference D under that latter process depends upon the precise relative radial location of the A-ring outer edge as compared with Janus' lower orbit semi-major axis, which is quantified here by means of a parameter p_o characterizing the efficiency of the effect of Janus' ring resonance. Under the combined effects of those two processes, depending on the value of p_o relative to a critical value $p_o \approx 19\%$, the system will evolve toward either a transition from horseshoe to tadpole orbit, or to a destruction of the co-orbital lock, after a few tens of Myears. Most recent observations of the A ring's outer edge by Spitale and Porco [Spitale, J.N., Porco, C.C., 2009. The Astronomical Journal 138, 1520–1528] tend to indicate that the future evolution will be a net tightening of the co-orbital system, until a tadpole situation will be reached after about 15 Myears. From a study of the past history of the co-orbital system, it is shown that such a scenario is compatible with a capture from free orbits to co-orbital horseshoe pattern some 25 Myears ago.

1. Introduction

Janus and Epimetheus are two co-orbital satellites of Saturn, that undergo "horseshoe" trajectories about their mass-averaged orbital state, as described by Dermott and Murray (1981a,b). In such a configuration, the co-orbital satellites are not librating closely around the L_4 or L_5 stable stationary points of the three-body system, but they librate over a large path including the L_3, L_4 and L_5 stationary points, on trajectories which appear as horseshoe shaped in the frame rotating with the satellites. Every 4 years, Janus and Epimetheus reach their mutual closest approach, and perform a swap between their lower and upper orbits. As shown by Tiscareno et al. (2009), under such a periodic change of their semi-major axes, both satellites undergo a spin libration about synchronous rotation, which is subsequently damped by internal friction under the effect of tidal torques. Noyelles (2010) and Robutel et al. (2011, 2012) performed further numerical studies that confirmed and extended Tiscareno et al.'s (2009) work. The main parameters of Janus and Epimetheus are given in Table 1. The aim of this paper is to discuss the effect of tides raised by Saturn on the long-term evolution of the horseshoe pattern. In Section 2, I show that the tidal torques exerted by the planet on the satellites tend to modify the magnitude of the separation between the semi-major axes of Janus and Epimetheus. In Section 3, I compare these effects to the ones produced by other processes contributing to the orbit evolution, and particularly the effects of torques produced by resonances with Saturn's rings. In Section 4, I discuss about the long-term evolution of the co-orbital system under the combined effects of those different mechanisms. Section 5 presents a summary and conclusions.

2. The effect of satellite tides on the co-orbital system

We will address here the dynamic evolution of the larger satellite Janus, although a similar description can be given for Epimetheus. Following the approach by Tiscareno et al. (2009), in this paper we will make the simplifying assumption that the orbit swap takes place instantaneously, with semi-major axis and mean motion changing from a_1 and n_1 (lower orbit) to a_2 and n_2 (higher orbit). In fact, as pointed out by Tiscareno et al. (2009) the orbit swap

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Table 1
Parameters of the Janus–Epimetheus system.

Parameter	Janus	Epimetheus
Average radius, R (km)[b]	89.5	58.1
Mass, m (10^{18} kg)[a]	1.897	0.526
Density, ρ (kg/m^3)[b]	632	641
Average mean motion, $\bar{n}$ (deg/day)[a]	518.292	518.292
n_1 (deg/day)[a]	518.3456	518.4865
n_2 (deg/day)[a]	518.2380	518.0976
$\delta n = n_1 - n_2$ (deg/day)	0.1076	0.3889
Average semi-major axis, $\bar{a}$ (km)[a]	151,450	151,450
$\delta a = a_2 - a_1$ (km)[c]	21.0	75.8
$(B-A)/C$[b]	0.100	0.280
ω_o (deg/day)[b]	284	488

[a] From Jacobson et al. (2008).
[b] From Tiscareno et al. (2009).
[c] From δn and from Eq. (1).

is not instantaneous but takes place over a period of several months, and the primary effect of this assumption is to maximize the amplitude of the free libration. Also we will ignore the effects of eccentricity, considering only guiding center trajectories. Throughout the paper we will note $\delta n = (n_1 - n_2)$ and $\delta a = (a_2 - a_1)$. Both δn and δa are positive, and from Kepler's third law they are related through

$$2\frac{\delta n}{n} = +3\frac{\delta a}{a} \tag{1}$$

From the values of δn displayed in Table 1, Eq. (1) permits to estimate δa as 21.0 km for Janus and 75.8 km for Epimetheus.

2.1. Simple case with no permanent bulge: tidal dissipation without libration

In the simplest approach, we will ignore the permanent bulge of Janus, but suppose that Janus is nonetheless rotating near synchronous rotation. This is the case referred to as the "naïve case" by Tiscareno et al. (2009). When Janus is on the lower orbit, we assume that just before the swap the rotation is synchronous and therefore the spin of the moon in inertial space is $\Omega = n_1$. Just after the swap the new semi-major axis and mean motion are $a_2 > a_1$, and $n_2 < n_1$, respectively, and therefore the satellite is now above synchronous height ($\Omega > n_2$). According to the standard theory of tides, the planet raises a tidal bulge on the satellite, which is offset from the satellite–planet line due to tidal dissipation, resulting in a relaxation of Ω toward synchronous rotation, together with a drift of the semi-major axis a, with the rates given by (e.g., Murray and Dermott, 1999):

$$\dot{\Omega} = -sign(\Omega - n)\frac{3k_{2S}}{2(2/5)Q_S}\left(\frac{M_P}{m_S}\right)\left(\frac{R_S}{a}\right)^3 n^2 \tag{2}$$

$$\dot{a} = sign(\Omega - n)\frac{3k_{2S}}{Q_S}\frac{M_P}{m_S}\left(\frac{R_S}{a}\right)^5 na \tag{3}$$

where k_{2S}, Q_S, m_S and R_S are the Love number, quality factor, mass and radius of the satellite, respectively, M_P is the mass of the planet, taken as 5.68×10^{26} kg for Saturn, $a = a_2$ is the semi-major axis, $n = n_2$ is the satellite mean motion.

For a small satellite, the Love number k_{2S} is adequately approximated by its value for a homogeneous rigid sphere (e.g., Peale, 1999):

$$k_{2S} \approx \frac{3}{19}\frac{\rho g R_S}{\mu} \tag{4}$$

where μ and ρ are the rigidity and the density of the satellite, respectively, while $g = Gm_S/R_S^2$ is its surface gravity acceleration, with G the gravitational constant.

Combining Eqs. (2)–(4), with Kepler's Law $n^2a^3 = G(M_p + m_s)$, with $m_s \ll M_p$, one gets (e.g., Peale, 1999):

$$\dot{\Omega} = -sign(\Omega - n)\frac{45}{76}\frac{\rho n^4 R_S^2}{\mu Q_S} \tag{5}$$

$$\dot{a} = -\frac{4}{5}\frac{R_S^2}{an}\dot{\Omega} \tag{6}$$

Standard values of rigidity and quality factor are $Q \approx 100$, and $\mu \approx 4 \times 10^9$ Nm^{-2} for a monolithic icy body, while for a fractured icy body Tiscareno et al. (2009) propose that both Q and μ would be smaller by an order of magnitude ($Q \approx 10$, $\mu \approx 4 \times 10^8$ Nm^{-2}). This permitted those authors to estimate the time necessary for despinning Janus from $(\Omega - n) = n_1 - n_2 = 0.108°$/day to zero as: $(\Omega - n)/\dot{\Omega} \approx 1$ year for solid ice, and ≈ 4 days for fractured ice. Therefore in both cases the time required is much less than the interval between orbital swaps, indicating that the spin will have relaxed toward synchronous rotation well before the next swap occurs.

Meanwhile, from Eq. (3), the semi-major axis increases under the effect of tides, with a rate given by Eq. (6).

The following swap will put Janus back to the lower orbit ($a = a_1$, $n = n_1$), and a similar tidal dissipation phenomenon will occur, except that this time Janus is under synchronous height ($\Omega < n_1$), the sign in Eqs. (2) and (3) is reversed, and under the effect of tidal dissipation the spin rate will increase while the semi-major axis will decrease. When the next encounter of Janus and Epimetheus occurs, the radial distance between both orbits has increased accordingly.

The effect of tidal dissipation will thus be to increase the semi-major axis when Janus is on its higher orbit, but to decrease it when Janus is on its lower orbit. As a consequence the distance between lower and higher orbit will systematically increase during the interval between two consecutive orbit swaps. The same phenomenon will occur for Epimetheus, although with a reduced efficiency (assuming the physical parameters are identical) because Epimetheus is a smaller satellite and thus less affected by tidal effects. If there were no other counterbalancing effect, the horseshoe pattern would evolve until the distance between the orbits of both moons would exceed a limit beyond which the horseshoe co-orbital situation is no longer possible.

2.2. Case including a permanent bulge and strong tidal dissipation

Although the preceding section gave the basic principle of the effect of tidal dissipation on the evolution of the Janus–Epimetheus horseshoe co-orbiting system, it ignored the effect of the permanent bulges of the satellites, and the more general situation is thus treated in detail in this section.

From an extensive analysis of images of the Cassini mission, Tiscareno et al. (2009) have shown that in reality Janus is not spherically symmetric, but possesses a significant permanent bulge, leading to additional torques raised by the planet on the satellite. Thereafter, more definitive shape results were also presented by Thomas (2010). The satellite may be modeled as a triaxial ellipsoid with principal moments of inertia A < B < C. Henceforth, the permanent bulge will be identified as the moon's long axis, which is the axis of smallest moment of inertia A, while axis C will be assumed perpendicular to the orbital plane.

Let us define as ζ the orientation of the permanent bulge in inertial space, and θ the orbital longitude of the moon. The spin libration angle, defined as the angle between the permanent bulge and the center of the planet, will therefore be $\psi = \zeta - \theta$. Differentiating twice, this gives $\ddot{\psi} = \ddot{\zeta} - \ddot{\theta}$. From Eqs. (1) and (6) this implies: