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Survey of Kozai dynamics beyond Neptune

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ABSTRACT

We study the Kozai dynamics affecting the orbital evolution of trans-neptunian objects being captured or not in MMR with Neptune. We provide energy level maps of the type (ω, q) describing the possible orbital paths from Neptune up to semimajor axis of hundreds of AU. The dynamics for non-resonant TNOs with perihelion distances, q, outside Neptune's orbit, a_N, is quite different from the dynamics of TNOs with $q < a_N$, already studied in previous works. While for the last case there are stable equilibrium points at $\omega = 0°$, 90°,180° and 270° in a wide range of orbital inclinations, for the former case it appears a family of stable equilibrium points only at a specific value of the orbital inclination, $i \sim 62°$, that we call critical inclination. We show this family of equilibrium points is generated by a mechanism analogue to which drives the dynamics of an artificial satellite perturbed by an oblate planet. The planetary system also generates an oscillation in the longitude of the perihelion of the TNOs with $i \sim 46°$, being Eris a paradigmatic case. We discuss how the resonant condition with Neptune modify the energy level curves and the location of equilibrium points. The asymmetric librations of resonances of the type 1:N generate a distortion in the energy level curves and in the resulting location of the equilibrium points in the phase space (ω, q). We study the effect on the Kozai dynamics due to the diffusion process in a that occurs in the Scattered Disk. We show that a minimum orbital inclination is required to allow substantial variations in perihelion distances once the object is captured in MMR and that minimum inclination is greater for greater semimajor axis.

1. Introduction

One of the open problems of the trans-neptunian region (TNR) is to find an explanation for the wide variety of orbits that the discovered trans-neptunian objects (TNOs) exhibit. In particular, those objects with perihelion outside Neptune's orbit and with high orbital inclinations and eccentricities that cannot be explained by the diffusive process the Scattered Disk Objects (SDOs) experience, nor by the past dynamical history of the planetary system (Tsiganis et al., 2005). It is known (Duncan et al., 1995; Gladman et al., 2002; Fernández et al., 2004) that TNOs with perihelion $q < 36$ AU are inside a chaotic region that generates a diffusion in semimajor axis enhancing aphelion distances. But the diffusion itself cannot decouple the perihelion from Neptune's region, then it is necessary to invoke another mechanism to explain the existence of high perihelion SDOs, also known as Detached Objects. The origin of these problematic orbits have given rise to theories involving passing stars (Ida et al., 2000), scattered planets (Gladman and Chan, 2006), tides from the star cluster where the Sun was formed (Brasser et al., 2006), a stellar companion (Gomes et al., 2006) and others. But, in order to avoid extra hypothesis it is necessary an abroad panorama of the dynamics generated by the planetary system itself in the TNR. Secular resonances, mean motion resonances (MMRs) and the Kozai resonance (KR) are dynamical mechanisms operating in the TNR and that could explain some eccentric orbits decoupled from encounters with Neptune (Fernández et al., 2004; Gomes et al., 2005; Gomes, 2011). The present work contributes with new results following that line of thinking.

Secular resonances do not affect the TNOs located beyond $a \sim 42$ (Knežević et al., 1991) and MMRs with Neptune do not generate themselves a substantial variation in the orbital elements, as for example the MMRs with Jupiter do (Gallardo, 2006b). The orbital variations observed when a particle is captured in MMR with Neptune are due to a secular dynamics inside the MMR, like Kozai dynamics. The Kozai dynamics, or more properly Kozai–Lidov dynamics, has its roots in a study by Lidov (1962) of the secular evolution of an artificial Earth's satellite perturbed by the Moon. Applying these ideas to the asteroid belt, Kozai (1962) developed an analytical approximation for the mean secular perturbing function due to Jupiter assumed in circular orbit, valid even for high inclination and high eccentricity asteroidal orbits. The conservation of the semimajor axis, as is usual in secular theories, but also the conservation of the parameter $H = \sqrt{1 - e^2} \cos i$, and the existence of an energy integral, $K(\omega, e)$, allowed the calculation of the energy level curves for this model showing large oscillations of e

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and i coupled with the argument of the perihelion, ω, around equilibrium points that appear for $H < 0.6$ at $\omega = 90°$ and 270°. These maps of energy level curves have axial symmetry with respect to $\omega = k90°$ being $k = 0,1,2,3,4$ and with respect to $i = 90°$. The orbital regime under oscillations of ω it is known since then as Kozai resonance (KR) and it provides an explanation to the existence of some asteroids in spite of their large orbital oscillations.

The conservation of H gives us the ultimate limits in e and i between which the particle's orbit can evolve assuming a secular dynamical regime, that means no close encounters with the planets, whether resonant or not. But the actual limits in e and i for a specific orbit are imposed by the orbital variations allowed by the energy level curve to which the particle is confined. We show in Fig. 1 the known population of TNOs with $q > 36$ AU, that means outside the diffusion region, in a diagram $H(i,e)$. These objects at present do not undergo encounters with Neptune and do not experience diffusion in semimajor axis, then their eccentric or very inclined orbits are due to the early history of the planetary system or to the secular dynamics acting at present. Each object could evolve along its corresponding H-constant curve allowing to increase or decrease its present perihelion distance, but the limits only can be defined when studying the energy level curve for each object.

Kozai (1985) modified his model to include also a mean motion resonant condition between the mean longitudes of Jupiter and the asteroid assuming a fixed value of the critical angle that characterizes the resonance, allowing to understand the KR inside a MMR (or MMR + KR). Energy level curves for resonant asteroids are very different than for the nonresonant case, and also are different the limits for the orbital oscillations. Moreover, for some resonances, new stable equilibrium solutions appear for ω different from the known 90° or 270°. These were called *asymmetric* equilibrium points some years after and they appear as stability islands between collision trajectories with the planets (Gronchi and Milani, 1999). Except for the case of the Jupiter's Trojans, the energy level diagrams exhibit, as in the nonresonant case, symmetry with respect to $\omega = k90°$.

Bailey et al. (1992) studied the orbital evolution of high inclination long period comets with a new semi-analytical averaging method that takes into account the perturbations of Jupiter assumed in circular orbit. The method is not based on any analytical series expansions but in calculating numerically the mean energy integral, doing it good for any set of orbital elements. They numerically computed energy level curves which long period comets should follow assuming no planetary close encounters or MMRs with Jupiter. That energy level curves showed stable equilibrium

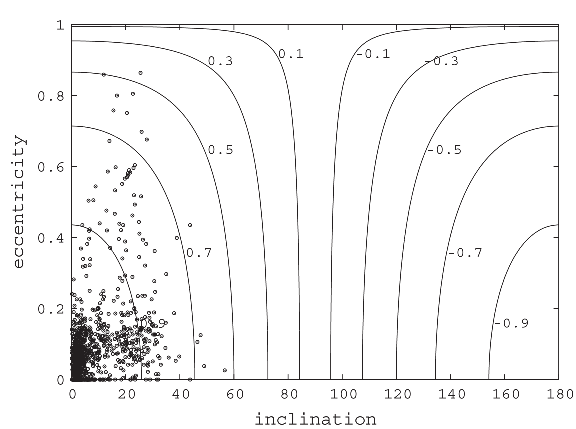

Fig. 1. Trajectories of constant H (labeled lines) and the known population of objects with $q > 36$ AU (open circles). All these objects could increase or decrease their q and i by Kozai dynamics following curves of constant H. Data corresponding to 986 objects from JPL, by January 2012.

oscillations around $\omega = k90°$ for cometary perihelion inside the planetary region. The principal result was that some comets have extreme perihelion oscillations which is clearly related to the origin of some sungrazing comets.

Thomas and Morbidelli (1996) followed an analogue method but including the four jovian planets in order to study the dynamics in the outer Solar System. The series of energy level curves they obtained give a very complete panorama of the perihelion behavior from the Neptune region down to the proximity of the Sun. They also obtained, as did Bailey et al. (1992), the same equilibrium oscillations around $\omega = k90°$ for the case of long period comets. Nevertheless, by their detailed study, it is clear now that the equilibrium islands at $\omega = 90°$, 270° with very small q are more stable than the ones at $\omega = 0°$, 180° which vanish in some circumstances due to the proximity with the collision curve with the planets, which is a curve in (ω,e) that implies orbital intersection between the particle and the planets for a given value of (a,H). Michel and Thomas (1996) followed the same method of Thomas and Morbidelli (1996) but including the effects of all the planets allowing the study of the dynamics of asteroids with $a < 2$ AU in NEA type orbit, showing by first time the Kozai dynamics associated with the terrestrial planets. Kinoshita and Nakai (1999, 2007) finally provided analytical solutions for the original Kozai model of an asteroid under an external perturber.

Fernández et al. (2004) and Gomes et al. (2005) in a series of numerical simulations showed the relevance that the interaction between MMRs and KR have for increasing significantly the perihelion distances of SDOs, from $q \sim 40$ AU to $q \sim 70$ AU. They found the KR generates large perihelion variations only when the TNO is captured in MMR with Neptune and preferably in resonances of the type 1:N. Gomes et al. (2005) also provide figures showing how the energy level curves are modified when a MMR condition is imposed.

Gallardo (2006a) using a classical expansion up to order 10 of the disturbing function due to Neptune analyzed how the KR is generated inside and outside a MMR with Neptune. It was found a new regime of the KR for orbits with perihelion outside Neptune in the far TNR: it appears a stable equilibrium point at $\omega = 90°$ and 270° but for a very particular value of the inclination, $i \sim 63°$, called critical inclination, which resembles the dynamical behavior of Molniya like artificial satellites. Curiously, Kuchner et al. (2002) had also detected notable eccentricity variations in fictitious particles with initial circular orbits with $i = 61°$ at $a = 43$ and $a = 45$ AU but with ω oscillating around 0° and 180°. Then, it is evident there is an interesting Kozai dynamics related to orbits with $i \sim 61$–63° in the TNR. Wan and Huang (2007) using the same classical expansion used in Gallardo (2006a) due to Neptune studied the effects of the KR inside the MMRs 2:3 and 1:2 providing energy level diagrams for that resonances, but they did not consider inclinations greater than 55°. We must take into account that Krasinsky (1972) proved the existence of the critical inclination in the framework of the restricted circular three-body problem for both situations: outer perturber and inner perturber, which are $i \sim 39°$ and $i \sim 63°$ respectively, for the case that the ratio of the semimajor axis of the inner body with respect to the outer body tends to zero.

Just for completeness, the Kozai dynamics was also studied in extrasolar systems as a natural extension of the original asteroidal problem, that means, the planet that generates the perturbation is outside the perturbed planet, see for example Innanen et al. (1997), Wiegert and Holman (1997), Holman et al. (1997), Mikkola and Tanikawa (1998) and Valtonen and Karttunen (2006, chapter 9), while the mutual perturbations of two-planet systems were only considered more recently in the context of the three body problem (Michtchenko et al., 2006; Libert and Tsiganis, 2009; Migaszewski and Goździewski, 2010). It is worth mention that a big deal of the analytical studies of the Kozai dynamics applied to satellites, small