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Öpik-type collision probability for high-inclination orbits

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ABSTRACT

The classical Öpik theory provides an estimate of the collision probability between two bodies on bound, heliocentric or planetocentric orbits under restrictive assumptions of: (i) constant eccentricity and inclination, and (ii) uniform circulation of the longitude of node and argument of pericenter. These assumptions are violated whenever either of the orbits has a large inclination with respect to the local Laplace plane or large eccentricity, and their motion is perturbed by an exterior (tidal) gravitational field of a planet or the Sun. In this situation, known as the Lidov–Kozai regime, the eccentricity and inclination values exhibit large and correlated oscillations. At the same time, the longitude of node and the argument of pericenter may have strongly nonlinear time evolution, with the latter being even bound to a small interval of values. Here we develop a new Öpik-type collision probability theory which is valid even for highly inclined and/or eccentric orbits of the projectile. We assume that the orbit of the target is circular and in the local Laplace plane. Such a generalized setting is necessary, as an example, to correctly estimate the terrestrial impact fluxes of sporadic micrometeoroids on high-inclination orbits (notably those from the toroidal source and the associated helion and anti-helion arcs).

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1. Introduction

Many problems in planetary science require to determine the collision probability of two bodies residing on the Keplerian orbits with the same focal point. Here we consider the important case in which the collision probability needs to be evaluated in a statistical sense for a large population of bodies. In this case, it is often useful if the probability is averaged over the secular orbital timescale.

The standard theory, used and extended by many researchers, was developed by Öpik (1951) (see also Öpik, 1976; Wetherill, 1967; Greenberg, 1982). In his original formulation, Öpik assumed that the target on a circular orbit is bombarded by a population of bodies on orbits with fixed eccentricity and inclination values.

Öpik's theory was generalized to the case of an eccentric orbit of the target by Wetherill (1967) and Greenberg (1982). A different generalized method was developed by Kessler and Cour-Palais (1978) (see also Kessler, 1981). This more geometrical approach based on the evaluation of the probability density distribution has found a number of applications in planetary science (e.g., Steel and Baggaley, 1985; Steel and Elford, 1986; Sykes, 1990).

In these standard collisional theories, the orbital eccentricity e and inclination i is assumed to be constant during the secular evolution cycle. This is appropriate for small e and i values, where e and i are roughly time-invariant. However, some problems in planetary science require a method that is valid for high eccentricities and/or high inclinations, where the effects of the Lidov–Kozai resonance can be important (e.g., Lidov, 1961, 1962; Kozai, 1962; Morbidelli, 2002).

For example, the dust particles released from long-period comets can be an important component of the zodiacal cloud. If so, it would be important to calculate their impact rates on the Earth (and relate the results to meteor observations), Earth-bound detectors and spacecrafts. Other applications can be found in studies of planetary impact rates in the early Solar System when small bodies were stochastically driven to high-e and -i orbits. In these examples, the secular evolution of orbits clearly violates the assumption of the standard Öpik theory, because e and i are affected by the Lidov–Kozai cycles.

Here we generalize the Öpik theory to account for the Lidov–Kozai cycles of high-i and -e orbits. After mathematical preliminaries in Section 2, we generalize the collisional probability theory in Section 3. In Section 3.3, we test the generalized theory by comparing it with direct N-body integrations of orbits. Conclusions are given in Section 4.

2. Mathematical preliminaries

We start by introducing mathematical concepts and notation that will be used throughout the paper. Assume a particle on an elliptic heliocentric orbit described using an osculating set of

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Keplerian elements: semimajor axis a, eccentricity e, inclination i, longitude of node Ω, argument of pericenter ω and true anomaly f. The angles i, Ω and ω are defined with respect to a chosen inertial frame (X,Y,Z).[1] The orbit intersects the (X,Y) reference plane in ascending and descending nodes, where $f = f_0 \equiv -\omega$ and $f = f_0 \equiv \pi - \omega$, respectively. Denote a' the heliocentric distance at either of the two intersections. Introduce a local reference basis $(\mathbf{e}_r, \mathbf{e}_\phi, \mathbf{e}_z)$ of three orthonormal vectors with the origin at the ascending or descending node, such that $\mathbf{e}_r$ is directed in the radial direction, $\mathbf{e}_\phi$ in the longitude direction and $\mathbf{e}_z$ along the Z axis.[2]

The heliocentric position vector $\mathbf{r}$ describing the elliptic orbit of the particle reads

$$\mathbf{r}(f) = r(f)[\mathbf{a}\cos(\omega + f) + \mathbf{b}\sin(\omega + f)], \tag{1}$$

with $r(f) = a\eta^2/(1 + e\cos f)$, $\eta^2 = 1 - e^2$, and unit vectors $\mathbf{a}^\mathrm{T} = (\cos\Omega, -\sin\Omega, 0)$ and $\mathbf{b}^\mathrm{T} = (-\cos i \sin\Omega, \cos i \cos\Omega, \sin i)$. At the ascending node we have $\mathbf{a} = \mathbf{e}_r$ and $\mathbf{b} = \cos i\mathbf{e}_\phi + \sin i\mathbf{e}_z$, while at the descending node $\mathbf{a} = -\mathbf{e}_r$ and $\mathbf{b} = -\cos i\mathbf{e}_\phi + \sin i\mathbf{e}_z$. Expanding $\mathbf{r}(f)$ near the origin in the local $(\mathbf{e}_r, \mathbf{e}_\phi, \mathbf{e}_z)$ system (i.e., near the respective nodal intersection with the (X,Y) reference plane), we obtain $\mathbf{r}(f) = a'\mathbf{e}_r + d\mathbf{r}$ with

$$d\mathbf{r} = a'\mathbf{A}_1\, df + \frac{a'}{2}\mathbf{A}_2\, df^2 + \mathcal{O}(df^3), \tag{2}$$

where df is infinitesimal increment of the true anomaly with respect to the intersection value f_0. Eq. (2) locally describes particle's elliptic orbit, with df being an affine parameter having values suitably close to zero. The first term is the crudest rectilinear approximation, while the second term describes the local curvature of the elliptic orbit. The first- and second-order vectorial coefficients read (upper sign for the ascending node intersection and lower sign for the descending node intersection)

$$\mathbf{A}_1 = \mp\frac{e\sin\omega}{P}\mathbf{e}_r + (\cos i\mathbf{e}_\phi \pm \sin i\mathbf{e}_z), \tag{3}$$

$$\mathbf{A}_2 = -2\left[1 - \frac{3}{2P} + \frac{\eta^2}{P^2}\right]\mathbf{e}_r - 2\frac{e\sin\omega}{P}(\pm\cos i\mathbf{e}_\phi + \sin i\mathbf{e}_z), \tag{4}$$

where $P = a\eta^2/a'$ and $\eta^2 = 1 - e^2$.

Consider now an observer moving on a circular heliocentric orbit with radius a' in the (X,Y) reference plane. Eq. (2) may be also used to describe its orbit near the respective nodal intersection with the eccentric orbit, with $\mathbf{A}_1^\mathrm{circ} = \mathbf{e}_\phi$ (henceforth also the apex direction), $\mathbf{A}_2^\mathrm{circ} = -\mathbf{e}_r$ and $df = df_\mathrm{circ}$, a differential in the observer's longitude. Denote V_circ the orbital velocity of the observer (given by the third Kepler law) and V the relative velocity of the particle with respect to the observer. It is convenient to introduce a scaled value υ of the relative velocity, namely $\upsilon = V/V_\mathrm{circ}$, and parametrize the complete relative vector $\boldsymbol{\upsilon}^\mathrm{T} = (\upsilon_r, \upsilon_\phi, \upsilon_z) = \upsilon(\cos b\sin\ell, \cos b\cos\ell, \sin b)$ $\upsilon_\phi, \upsilon_z) = \upsilon(\cos b\sin\ell, \cos b\cos\ell, \sin b)$ with a longitude ℓ and a latitude b of the radiant seen by the observer (henceforth, ℓ is measured from the apex direction and increases toward local radial direction in our notation). We also note that our choice makes $\boldsymbol{\upsilon}$ point *toward* the radiant from which the observer sees the particle impact.

The velocity components $(\upsilon_r, \upsilon_\phi, \upsilon_z)$ may be easily obtained from the linear term in (2), namely by using

$$\boldsymbol{\upsilon} = \mathbf{e}_\phi - \frac{1}{V_\mathrm{circ}}\left(\frac{d\mathbf{r}}{dt}\right)_{f=f_0} = \mathbf{e}_\phi - \mathbf{A}_1\sqrt{P}. \tag{5}$$

We thus obtain

$$e\cos\omega = \pm(P - 1), \tag{6}$$

$$e\sin\omega = \pm\sqrt{P}\upsilon_r, \tag{7}$$

$$\sqrt{P}\cos i = 1 - \upsilon_\phi, \tag{8}$$

$$\sqrt{P}\sin i = \mp\upsilon_z, \tag{9}$$

where the upper sign holds for the ascending node intersection and the lower sign for the descending node intersection. Here the first formula (6) is simply the geometric condition of intersection at heliocentric distance a' (as stated above), and the next three formulas (7)–(9) specify the radiant location and impact velocity (in units of V_circ). Obviously, our $(\upsilon_r, \upsilon_\phi, \upsilon_z)$ are closely related, in fact identical, to the standard velocity components (U_x, U_y, U_z) introduced in the Öpik theory (see, e.g., Öpik, 1951, 1976).

Finally, it will be useful to rewrite beforehand Eq. (6) using the non-singular variables $k = e\cos\omega$ and $h = e\sin\omega$ and parameter $\alpha = a'/a$. In the (k,h) plane the nodal intersection condition (6) reads

$$\left(k \pm \frac{\alpha}{2}\right)^2 + h^2 = 1 - \alpha + \frac{\alpha^2}{4}, \tag{10}$$

which is simply an equation of a circle displaced by $\pm\alpha/2$ on the k-axis for the ascending, resp. descending, node and radius equal to $\sqrt{1 - \alpha + \alpha^2/4}$.

3. Öpik collision probability approach

In the Öpik approach, the collision probability of a particle with a target is composed of two independent parts: (i) probability P_1 that during the secular cycle of the particle orbital elements its heliocentric node is close to the target's circular orbit (such that their distance can be small enough), and (ii) probability P_2 that the target is close to the nodal intersection of the particle orbit. A product of these statistically independent partial probabilities provides the total probability of impact per revolution of the particle: $P = P_1P_2$. Dividing this value by the orbital period of the particle then yields total probability per unit of time (this is because we assume an equilibrium distribution of particles along the impacting orbit). Obviously, in this way the resulting collision probability is a long-term averaged value or, equivalently, a population averaged value for a large population of particles in steady-state.

Because we keep the assumption of the circular motion of the target and the rectilinear representation of the particle motion near the nodal configurations (first term in Eq. (2)), analysis of P_2 is the same as in Öpik (1951). In particular, assuming the target with radius R on a circular heliocentric orbit with radius a_circ, we have

$$P_2(a,e,i) = \frac{R}{4a_\mathrm{circ}}\sqrt{\frac{3 - T(a,e,i)}{2 - F(a,e,i)}}, \tag{11}$$

with

$$T(a,e,i) = \frac{a_\mathrm{circ}}{a} + 2\sqrt{\frac{a}{a_\mathrm{circ}}}\eta\cos i, \tag{12}$$

$$F(a,e,i) = \frac{a_\mathrm{circ}}{a} + \frac{a}{a_\mathrm{circ}}\eta^2\cos^2 i. \tag{13}$$

However, to compute P_1, Öpik's assumed constant values of e and i and uniform circulation of ω. This is an acceptable approximation for low inclination and low eccentricity orbits, but it fails when either of or both these elements are large. Our goal is to extend determination of P_1 for orbits with arbitrary inclination and eccentricity values.

[1] We assume $i \neq 0$, otherwise a non-singular set of orbital elements would be needed. In order to keep a close similarity in notation to the works of Öpik (1951) and Wetherill (1967) we only consider the non-planar case.

[2] Note that the $\mathbf{e}_r$ and $\mathbf{e}_\phi$ vectors at the descending node are opposite to their values in the descending node, and vice versa, in our definition.

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