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Analytic perturbative theories in highly inhomogeneous gravitational fields

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ABSTRACT

Orbital motion about irregular bodies is highly nonlinear due to inhomogeneities in the gravitational field. Classical theories of motion close to spheroidal bodies cannot be applied as for inhomogeneous bodies the Keplerian forces do not provide a good approximation of the system dynamics.

In this paper a closed form, analytical method for developing the motion of a spacecraft around small bodies is presented, for the so called fast rotating case, which generalize previous results to second order, arbitrary degree, gravitational fields. Through the application of two different Lie transformations, suitable changes of coordinates are found, which reduce the initial nonintegrable Hamiltonian of the system into an integrable one plus a negligible, perturbative remainder of higher degree. In addition, an explicit analytical formulation for the relegated, first and second order, arbitrary degree Hamiltonian for relatively high altitude motion in any inhomogeneous gravitational field is derived in closed-form. Applications of this algorithm include a method for determining initial conditions for frozen orbits around any irregular body by simply prescribing the desired inclination and eccentricity of the orbit. This method essentially reduces the problem of computing frozen orbits to a problem of solving a 2-D algebraic equation. Results are shown for the Asteroid 433-Eros.

1. Introduction

The motion of bodies subject to non-Keplerian gravitational fields is a classical subject of research in the context of celestial mechanics. In recent years this type of research has become important to future planned missions of spacecraft to the Moon and asteroids in addition to asteroid deflection missions such as the European Space Agency's "Don Quijote" concept (Carnelli and Gálvez, 2006). Research undertaken in this area has studied the effect of the Earth's inhomogeneous gravitational field on the motion of natural and artificial satellites, that is, artificial satellite theory for small and moderate eccentricities (Deprit, 1970). More recent studies have researched the effects on motion of the inhomogeneous gravitational field of other Solar System bodies, including the Moon (Abad et al., 2009) and asteroids (San-Jaun et al., 2004). The analysis of spacecraft motion about these bodies is particularly challenging as they typically feature shapes and density distributions more irregular than those of planets. Such irregularities break symmetries and require more complicated analytical expressions for their description which increases the complexity involved in such studies.

Numerical methods are today widely used to study the trajectories of objects orbiting specific irregular bodies (Fahnestock and Scheeres, 2008) or for finding stability criteria (Lara and Scheeres, 2002). Disadvantages of these methods are that they can be highly computational and require a complete re-design for each different body. Analytical methods, by contrast, have the potential to rapidly identify useful natural motions for general bodies with inhomogeneous gravitational fields. Furthermore, they can provide a full dynamical picture of the motion around irregular bodies that can be used to search and study particular classes of useful orbits. However, current analytical methods are only used in a limited and semi-numerical way (meaning that analytical expansions constitute the first step in such studies, which are then typically carried out from a numerical standpoint (Scheeres et al., 1998)). The main drawbacks of these methods is that their application in the case of highly inhomogeneous bodies requires extensive symbolic computations involving algebraic manipulations, and that they are usually restricted to a certain range of eccentricities due to series convergence. Analytical studies on inhomogeneous gravitational fields have been, so far, limited to low degree gravity fields (Palacián, 2002, 2007; San-Juan et al., 2002, 2004), thus restricting the results to a class of bodies for which the dynamics is dominated by a few coefficients (e.g. oblateness or ellipticity).

In this paper a closed form (i.e. without using series expansion in the eccentricity), analytical, perturbative theory of motion around inhomogeneous bodies is presented, generalized to second order, arbitrary degree gravity fields.

Using Deprit and Palacián's relegation algorithm (Palacián, 1992) and a Delaunay normalization, suitable canonical

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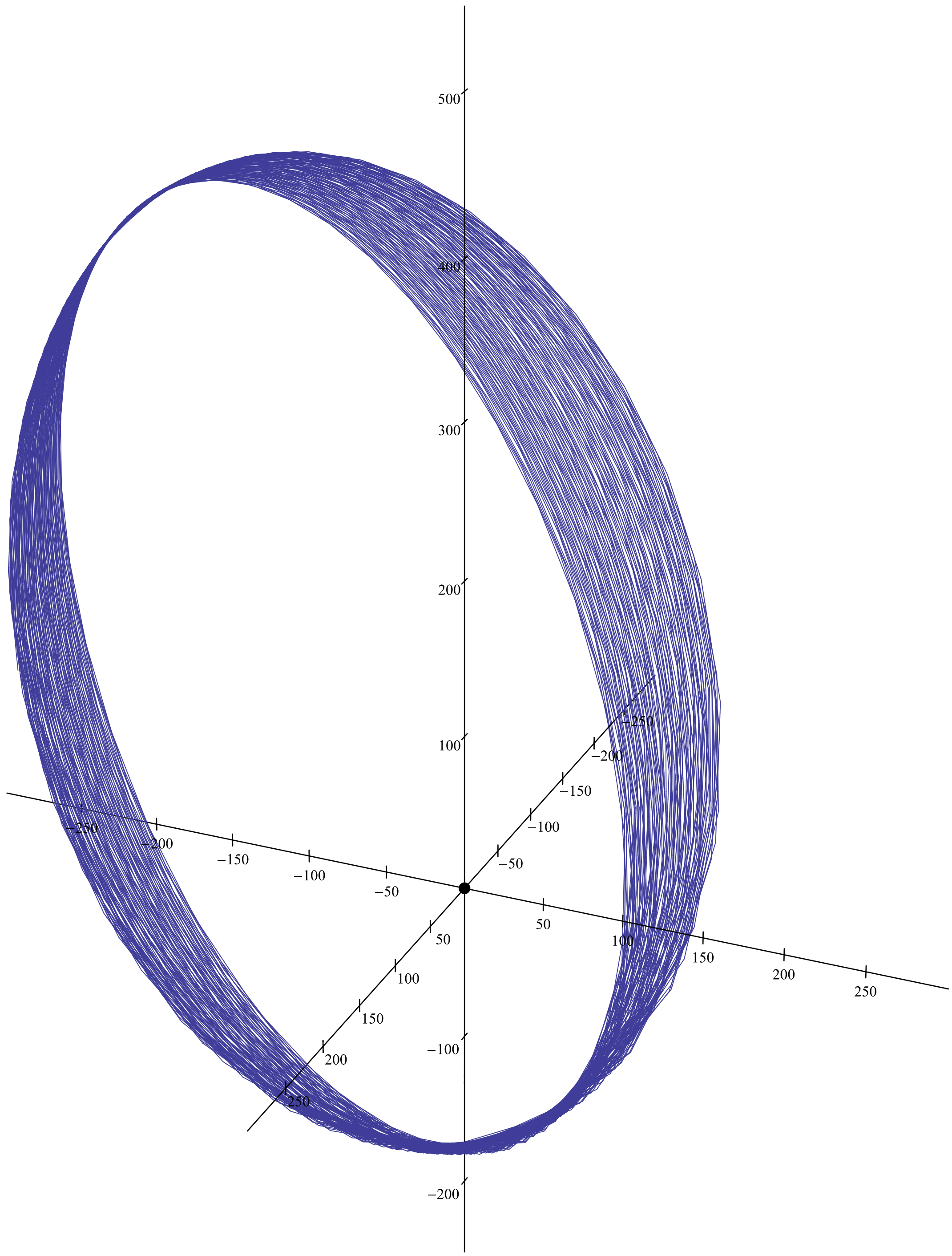

Fig. 1. The resulting frozen orbit for E_0 = 0.5, I_0 = 1.1 and $g_0 = -\frac{\pi}{2}$ for 5 years.

action-angle variables are found, which reduce the initial non-integrable Hamiltonian into an integrable one plus a negligible, perturbative remainder. The method can be used to find useful orbits for space mission applications such as frozen orbits. Moreover, frozen orbits are orbits with no secular perturbations in the inclination,P argument of pericenter, and eccentricity (Brouwer, 1959). These orbits are periodic orbits, except for the orbital plane of precession, and are therefore called frozen. In particular, this paper extends previous work by:

- Formulating the inhomogeneous gravitational potential generated by any inhomogeneous body in polar-nodal coordinates.
- Including arbitrary degree gravitational coefficients, instead of limiting the study to 2nd degree coefficients.
- Stating the explicit analytical formulation for the closed-form averaged with respect to the argument of node, second order, arbitrary degree Hamiltonian of any inhomogeneous gravitational field.
- Obtaining a resulting Hamiltonian which accounts for the presence of the angular momentum, in contrast to the trivially integrable Hamiltonian of San-Juan et al. (2002) which only accounts for the argument of node. Again, this previous result was only possible by considering a Hamiltonian with 2nd degree coefficients.