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DEM simulation of rotation-induced reshaping and disruption of rubble-pile asteroids

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ABSTRACT

A Soft-Sphere Discrete Element Method (SSDEM) is used to simulate the rotational reshaping and disruption of cohesionless self-gravitating granular aggregates (as a representation of "rubble-pile" asteroids). Aggregates with spherical and ellipsoidal shapes are subjected to impulsive increments of their angular velocity to initiate a reshaping process leading up to the disruption of the aggregate. Internal stress fields are monitored during the process as well as critical angular velocities to initiate reshaping. In addition, the time evolution of other parameters such as filling fraction, angle of friction, mechanical energy, yield stress, semi-axes, density and mass dependence are also analysed. Several predictions from continuum theory are recovered in our simulations, in addition to further insight into the process by which cohesionless rubble piles can deform. Fundamentally different outcomes are found for frictionless grains and grains with surface friction modelled, verifying the importance of including such models in granular simulations. We find that the initiation of shape deformation is most consistently described by a Drucker–Prager failure criterion, which also provides an independent measure of the effective friction angle of our self-gravitating pile. Insight is also gained into the energetics of deformation, with most of the kinetic energy loss going into the deformation of the rubble pile, and a smaller component being internally dissipated. Finally, with this work we want to compare this computational approach with the theoretical predictions and, if possible, to mutually validate them.

1. Introduction

Given the current understanding that asteroids can be in fact rubble-piles (Davis et al., 1982, 1985; Fujiwara et al., 2006; Miyamoto et al., 2007) held together by gravitational and cohesive forces (Scheeres et al., 2010), it is necessary to understand their internal mechanics if we are to understand how they have evolved to show the variety of sizes, shapes and configurations that we can attest today. One of the possible reasons for these varied features could be the deformation and subsequent fission of the asteroids due to persistent rotational acceleration from the YORP effect (Rubincam, 2000). Holsapple (2001, 2004, 2010) has provided a theoretical approach to investigate the deformation/reshaping of rotating asteroids in the Solar System and Walsh et al. have provided some results from simulations using a Hard-Sphere DEM code (Walsh et al., 2008). However, there are some additional questions that can be answered by using a Soft-Sphere DEM code to simulate a "rubble-pile" asteroid. In particular, in this paper we deal with those related to internal stress fields and energy loss due to effective friction, which comprises surface friction and geometrical constraints. The changes in shape, bulk density and total energy as their total angular momentum is incremented are also calculated. In addition to our other studies, one specific question which this paper addresses is the apparent disagreements between the theoretical calculations of Holsapple and the numerical simulations of asteroid fission presented by Walsh et al. (2008), as discussed at the end of Holsapple's (2010) paper. We show that through the inclusion of particle–particle surface friction we are able to perform simulations that agree with the Holsapple theory, and through its neglect we are able to show simulations that agree with Walsh et al. Thus, this paper also helps address this outstanding issue reported earlier in the literature. It must be noted that, as indicated in the abstract, in our simulations we have not implemented cohesive forces of any sort.

The main difference between the simulations reported here and theoretical approaches is that simulations can capture the discrete nature of the systems, as that is the nature of a DEM simulation. However, it is difficult to capture the time-scales that real asteroids need to acquire the angular velocities needed for reshaping or disruption. Regardless of these shortcomings, both approximations have proven to be valuable tools given the impossibility to form a real asteroid and experiment on it. Keeping that in mind, it is also

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our purpose to mutually validate the theoretical predictions for self-gravitating aggregates and this computational method. A thorough comparison as such has not been carried out before and it is our belief that this is the first step that needs to be taken if theoretical and computational approaches are to be accepted as valid in micro-gravity environments. If both approaches produce agreeing results that will mean that SSDEM codes, and the already developed theory, could be used confidently to do research on self-gravitting granular aggregates.

The use of a SSDEM code that specifically uses a Mid-Step Velocity Verlet algorithm to integrate the equations of motion (Swope et al., 1982; Allen and Tildesley, 1989), will allow us to have access to all dynamical variables of the granular systems at any instant of the simulation. This will allow us to monitor derived quantities such as stress fields, filling/volume/packing fraction, energy (potential, kinetic and elastic), changes in shape and the related moments of inertia and their associated ellipsoid axes. This in turn will help us to answer some questions that have not been fully answered through theory or previous simulations. Do asteroids slowly reshape until they acquire enough angular velocity as to shed material? Is the critical angular velocity for reshaping different for aggregates with and without particle–particle surface friction? If so, how different? Starting from a perfectly spherical configuration, do bodies always go oblate or can they go prolate? When a self-gravitating aggregate deforms under shear, does it increase its volume? These are some of the questions that we investigate in this research. Specifically, we investigate the re-shaping process and the internal stress that the primary undergoes as a result of secular increases in angular momentum and before fission occurs.

2. SSDEM simulation

2.1. Short range interactions

The simulation program that is used for this research applies a Soft-Sphere Discrete Element Method (Cundall, 1971; Cundall and Hart, 1992; Allen and Tildesley, 1989; Sánchez and Scheeres, 2011b) to simulate a self-gravitating granular aggregate. This means that individual particles interact through a soft-repulsive potential when in contact. To ease the calculations, particles are modelled as spheres, but other shapes are possible (Langston et al., 2004). This method considers that two particles are in contact when they overlap. When this happens, normal and tangential contact forces are calculated (Herrmann and Luding, 1998). The calculation of the normal forces between colliding particles is modeled by a linear spring and a dashpot. The elastic force is modelled as

$$\vec{\mathbf{f}}_e = k_n \xi \hat{\mathbf{n}}, \tag{1}$$

and the damping force as:

$$\vec{\mathbf{f}}_d = -\gamma_n \dot{\xi} \hat{\mathbf{n}}, \tag{2}$$

Then the total normal force is calculated as $\vec{\mathbf{f}}_n = \vec{\mathbf{f}}_e + \vec{\mathbf{f}}_d$. In these equations k_n is the elastic constant, ξ is the overlap of the particles, γ_n is the damping constant (related to the dashpot), $\dot{\xi}$ is the rate of deformation and $\hat{\mathbf{n}}$ is the vector joining the centres of the colliding particles. This dashpot models the energy dissipation that occurs during a real collision.

The tangential component of the contact force models surface friction, static and dynamic. This is calculated by placing a linear spring at the contact point, attached to both particles, at the beginning of the collision (Herrmann and Luding, 1998; Silbert et al., 2001), producing a restoring frictional force $\vec{\mathbf{f}}_t$. To explain this, let $\vec{\mathbf{D}}$ be the total tangential elongation of this spring during the collision, then whilst there is a static contact, it is defined as:

$$\vec{\mathbf{D}} = \int_0^t \vec{\mathbf{v}}_t dt \tag{3}$$

In the code and during the static contact:

$$\vec{\mathbf{D}} = \sum \vec{\mathbf{v}}_t \delta t \tag{4}$$

where $\vec{\mathbf{v}}_t$ is the instantaneous relative tangential velocity of contact of the points of contact and δt is the time step for the integration of the equations of motion. Also, a tangential unit vector $\hat{\mathbf{t}}$ can be defined through $\vec{\mathbf{v}}_t$. This unit vector is (by construction) parallel to the circular plane contained by the intersection of the spherical surfaces of the two particles (for a more detailed explanation of the model see references Sánchez and Scheeres, 2011b; Herrmann and Luding, 1998). Then, a restoring tangential force is:

$$\vec{\mathbf{f}}_t = -k_t \vec{\mathbf{D}} \tag{5}$$

The magnitude of the elongation of this tangential spring is truncated in order to satisfy the local Coulomb yield criterion $|\vec{\mathbf{f}}_t| \leqslant \mu |\vec{\mathbf{f}}_n|$. Thus:

$$\vec{\mathbf{f}}_{fr} = \begin{cases} -k_t \vec{\mathbf{D}} & \text{if} \quad |\vec{\mathbf{f}}_t| \leqslant \mu |\vec{\mathbf{f}}_n| \ \text{(sticking)}, \\ -\mu |\vec{\mathbf{f}}_n| \hat{\mathbf{t}} & \text{if} \quad \mu |\vec{\mathbf{f}}_n| < |\vec{\mathbf{f}}_t| \ \text{(sliding)}. \end{cases} \tag{6}$$

This provides an effective stick–slip friction force between two particles in contact. Now, the springs in the normal and tangential are directions two simple harmonic oscillators and, for simplicity, we want to design this contact model system so that the normal direction spring will be back to its equilibrium position at the same time as the tangential spring. In order to achieve this, we take $k_t = (2/7)k_n$ (Silbert et al., 2001; Shäfer et al., 1996; Walton, 1995); in 2D $k_t = (1/3)k_n$. According to Mindlin (1949), the ratio k_t/k_n is between 2/3 and 1 for most materials and depends on the Poison ratio. However, Silbert et al. (2001) reports that simulations with different values for this ratio did not produce any significant change except for the average kinetic energy.

The simulations presented here do not implement a tangential dashpot ($\gamma_t = 0$); Silbert et al. (2001) found that its introduction only quickened the approach to a steady state by draining more energy, leaving all the other quantities unchanged. Therefore, there is no lose of generality by leaving it out of our calculations.

In order to speed up the calculation, the available space is divided in cubical cells and a link-list (Allen and Tildesley, 1989) is used to detect possible particle–particle contacts. This reduces the order of complexity of the contact detection algorithm from $O(N^2)$ to $O(N\log(N))$.

2.2. Long-range interactions

Apart from the contact forces (short-range interactions), the size of the bodies that are modelled and the absence of a strong-external gravitational field makes it indispensable to calculate also gravitational forces (long-range interactions). The direct calculation is computationally very expensive as its complexity is of order $O(N^2)$. To deal with it, tree codes and Fast Multipole Methods (FMM) have been developed. In the same vein, we have developed a method that divides the available space in cubical *CELLS*, larger than those used for the link-list, and then treats the particles inside each one of them as one particle. The order of complexity of this method is still $O(N^2)$, but it is at least one order of magnitude faster than the direct calculation. In addition, the implementation of such a method is, in our opinion, much faster and easier than that of a tree code or an FFM. Let us suppose two *CELLS*, C_A and C_B, contain N_A and N_B particles respectively. Then the jth particle in A will