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Spin axis evolution of two interacting bodies

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ABSTRACT

We consider the solid–solid interactions in the two body problem. The relative equilibria have been previously studied analytically and general motions were numerically analyzed using some expansion of the gravitational potential up to the second order, but only when there are no direct interactions between the orientation of the bodies. Here we expand the potential up to the fourth order and we show that the secular problem obtained after averaging over fast angles, as for the precession model of Boué and Laskar [Boué, G., Laskar, J., 2006. Icarus 185, 312–330], is integrable, but not trivially. We describe the general features of the motions and we provide explicit analytical approximations for the solutions. We demonstrate that the general solution of the secular system can be decomposed as a uniform precession around the total angular momentum and a periodic symmetric orbit in the precessing frame. More generally, we show that for a general *n*-body system of rigid bodies in gravitational interaction, the regular quasiperiodic solutions can be decomposed into a uniform precession around the total angular momentum, and a quasiperiodic motion with one frequency less in the precessing frame.

1. Introduction

We consider here two rigid bodies orbiting each other. The main purpose of this work is to determine the long term evolution of their spin orientation and to a lower extent, the orientation of the orbital plane. Examples of such systems are binary asteroids or a planet with a massive satellite.

If the two bodies are spherical, then the translational and the rotational motions are independent (e.g. Duboshin, 1958). In that case, the orbit is purely Keplerian and the proper rotation of the bodies are uniform. General problems with triaxial bodies are more complicated, and usually non-integrable. Even formal expansions of the gravitational potential or the proof of their convergence can be an issue (Borderies, 1978; Paul, 1988; Tricarico, 2008). In some cases, especially for slow rotations close to low order spin–orbit resonances, the spin evolution of rigid bodies of irregular shape can be strongly chaotic (Wisdom et al., 1984; Wisdom, 1987), but we will not consider this situation in the present paper where we focus on regular and quasiperiodic motions.

Stationary solutions of spin evolution are known in the case of a triaxial satellite orbiting a central spherical planet (Abul'naga and Barkin, 1979). In their paper, Abul'naga and Barkin used canonical coordinates, based on the Euler angles, to set the orientation of the satellite. On the contrary, in 1991, Wang et al. also studied relative equilibria but with a vectorial approach that enabled them to analyze easily the stability of those solutions. For a review of different formalisms that can be used in rigid body problems, see Borisov and Mamaev (2005).

The vectorial approach turned out to be also powerful for the study of relative equilibria of two triaxial bodies orbiting each other (Maciejewski, 1995). General motions of this problem were studied by Fahnestock and Scheeres (2008) in the case of the typical binary asteroid system called 1999 KW4. For that, the authors expanded the gravitational potential up to the second order only. In this approximation, there is no direct interaction between the orientation of the two bodies. Ashenberg gave in 2007 the expression of the gravitational potential expanded up to the fourth order but did not study the solutions.

In Boué and Laskar (2006) we gave a new method to study the long term evolution of solid body orientations in the case of a star–planet–satellite problem where only the planet is assumed to be rigid. This method used a similar vectorial approach as Wang et al. (1991), plus some averaging over the fast angles. We showed that the secular evolution of this system is integrable and provided the general solution.

In the present paper, we show that the problem of two triaxial bodies orbiting each other is very similar to the star–planet–satellite problem and thus can be treated in the same way.

In Section 2, we compute the Hamiltonian governing the evolution of two interacting rigid bodies. The gravitational potential is expanded up to the fourth order and averaged over fast angles. The resulting secular Hamiltonian is a function of three vectors only: the orbital angular momentum and the angular momenta of the two bodies.

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In a next step (Section 3), we show that the secular problem is integrable but not trivially (i.e. it cannot be reduced to a scalar first order differential equation that can be integrated by quadrature). The general solution is the product of a uniform rotation of the three vectors (global precession around the total angular momentum) by a periodic motion (nutation). We prove also that in a frame rotating with the precession frequency, the nutation loops described by the three vectors are all symmetric with respect to a same plane containing the total angular momentum. We then derive analytical approximations of the two frequencies of the secular problem with their amplitudes. These formulas need averaged quantities that can be computed recursively. However we found that the first iteration already gives satisfactory results.

In Section 5, we consider the general case of a n-body system of rigid bodies in gravitational interaction, and we demonstrate that the regular quasiperiodic solutions of these systems can, in a similar way, be decomposed into a uniform precession, and a quasiperiodic motion in the precessing frame.

Finally, we compare our results with those of Fahnestock and Scheeres (2008) on the typical binary asteroid system 1999 KW4. We show that their analytical expression of the precession frequency corresponds to the simple case of a point mass orbiting an oblate body treated in Boué and Laskar (2006). We then integrate numerically from the full Hamiltonian, an example of a doubly asynchronous system where the Fahnestock and Scheeres (2008) expression of the precession frequency does not apply. We compare the results with the output of the averaged Hamiltonian and with our numerical approximation and show that they are in good agreement.

2. Fundamental equations

We are considering a two rigid body problem in which the interaction is purely gravitational with no dissipative effects. Let m_1 and m_2 be the masses of the two solids. Hereafter the mass m_2 is called the satellite or the secondary and the mass m_1 the primary. It should be stressed that this notation does not imply any constraint on the ratio of the masses which can even be equal to one.

The configuration of the system is described by the position vector $\mathbf{r}$ of the satellite barycenter relative to the primary barycenter and their orientation expressed in an invariant reference frame. The orientations are given by the coordinates of the principal axes $(\boldsymbol{I}_1, \boldsymbol{J}_1, \boldsymbol{K}_1)$ and $(\boldsymbol{I}_2, \boldsymbol{J}_2, \boldsymbol{K}_2)$ in which the two inertia tensors, respectively $\mathcal{I}_1$ and $\mathcal{I}_2$, of the primary and of the secondary are diagonal $[\mathcal{I}_1 = \mathrm{diag}(A_1, B_1, C_1)$ and $\mathcal{I}_2 = \mathrm{diag}(A_2, B_2, C_2)]$.

The Hamiltonian of this problem can be split into

$$\mathcal{H} = H_T + H_E + H_I, \tag{1}$$

where H_T is the Hamiltonian of the free translation of the reduced point mass $\beta = m_1 m_2/(m_1+m_2)$, H_E describes the free rigid rotation of the two bodies and H_I contains the gravitational interaction.

The Hamiltonian of the free point mass is

$$H_T = \frac{\tilde{\mathbf{r}}^2}{2\beta}, \tag{2}$$

where $\tilde{\mathbf{r}} = \beta\dot{\mathbf{r}}$ is the conjugate momentum of $\mathbf{r}$.

Let $\boldsymbol{G}_1$ and $\boldsymbol{G}_2$ be respectively the angular momentum of the primary and of the satellite. The Hamiltonian of the free rotation is

$$H_E = \frac{{}^t\boldsymbol{G}_1\mathcal{I}_1^{-1}\boldsymbol{G}_1}{2} + \frac{{}^t\boldsymbol{G}_2\mathcal{I}_2^{-1}\boldsymbol{G}_2}{2}, \tag{3}$$

where the superscript ^{t} in ${}^t\mathbf{x}$ or ${}^tA$ denotes the transpose of any vector $\mathbf{x}$ or matrix A. It can be expressed in terms of the principal bases of the two bodies as follows

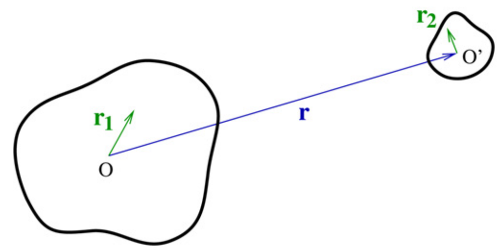

Fig. 1. Coordinates definition.

$$H_E = \frac{(\boldsymbol{G}_1\cdot\boldsymbol{I}_1)^2}{2A_1} + \frac{(\boldsymbol{G}_1\cdot\boldsymbol{J}_1)^2}{2B_1} + \frac{(\boldsymbol{G}_1\cdot\boldsymbol{K}_1)^2}{2C_1} + \frac{(\boldsymbol{G}_2\cdot\boldsymbol{I}_2)^2}{2A_2} + \frac{(\boldsymbol{G}_2\cdot\boldsymbol{J}_2)^2}{2B_2} + \frac{(\boldsymbol{G}_2\cdot\boldsymbol{K}_2)^2}{2C_2}. \tag{4}$$

The interaction between the two solid bodies is the following double integral

$$H_I = -\iint \frac{\mathcal{G}\,dm_1\,dm_2}{\|\mathbf{r}+\mathbf{r}_2-\mathbf{r}_1\|}, \tag{5}$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are respectively computed relative to the primary and satellite barycenters (cf. Fig. 1) and describe the two volumes. This part of the Hamiltonian can be expanded in terms of Legendre polynomials and will be written as a function of $(\mathbf{r}, \boldsymbol{I}_1, \boldsymbol{J}_1, \boldsymbol{K}_1, \boldsymbol{I}_2, \boldsymbol{J}_2, \boldsymbol{K}_2)$ in Section 2.3.

2.1. Equations of motion

The full Hamiltonian is written in the non-canonical coordinates $(\mathbf{r}, \tilde{\mathbf{r}}, \boldsymbol{I}_1, \boldsymbol{J}_1, \boldsymbol{K}_1, \boldsymbol{G}_1, \boldsymbol{I}_2, \boldsymbol{J}_2, \boldsymbol{K}_2, \boldsymbol{G}_2)$. Thus, although the components $(\mathbf{r}, \tilde{\mathbf{r}})$ keep the standard symplectic structure $(\boldsymbol{I}_1, \boldsymbol{J}_1, \boldsymbol{K}_1, \boldsymbol{G}_1)$ on the one hand and $(\boldsymbol{I}_2, \boldsymbol{J}_2, \boldsymbol{K}_2, \boldsymbol{G}_2)$ on the other hand possess the Euler–Poisson structure which leads to the following equations of motion (Borisov and Mamaev, 2005)

$$\begin{aligned}
&\dot{\mathbf{r}} = \nabla_{\tilde{\mathbf{r}}}\mathcal{H}, \qquad \dot{\tilde{\mathbf{r}}} = -\nabla_{\mathbf{r}}\mathcal{H},\\
&\dot{\boldsymbol{G}} = \nabla_{\boldsymbol{I}}\mathcal{H}\times\boldsymbol{I} + \nabla_{\boldsymbol{J}}\mathcal{H}\times\boldsymbol{J} + \nabla_{\boldsymbol{K}}\mathcal{H}\times\boldsymbol{K} + \nabla_{\boldsymbol{G}}\mathcal{H}\times\boldsymbol{G},\\
&\dot{\boldsymbol{I}} = \nabla_{\boldsymbol{G}}\mathcal{H}\times\boldsymbol{I}, \qquad \dot{\boldsymbol{J}} = \nabla_{\boldsymbol{G}}\mathcal{H}\times\boldsymbol{J}, \qquad \dot{\boldsymbol{K}} = \nabla_{\boldsymbol{G}}\mathcal{H}\times\boldsymbol{K}.
\end{aligned} \tag{6}$$

We choose these non-canonical coordinates instead of symplectic ones because of the simplicity of the resulting equations which already resemble equations of precession.

2.2. First simplification

In the previous paragraphs, the Hamiltonian contains the three vectors of the principal frame $(\boldsymbol{I}, \boldsymbol{J}, \boldsymbol{K})$ of each body. Nevertheless, only two vectors per solid are necessary insofar as the third can be expressed as the wedge product of the other two. We choose to keep $\boldsymbol{I}$ and $\boldsymbol{K}$.

The Hamiltonian of the free rotation of the two rigid bodies can be rewritten as follows

$$\begin{aligned}
H_E = {}& \frac{\boldsymbol{G}_1^2}{2B_1} + \frac{\boldsymbol{G}_2^2}{2B_2} + \left(\frac{1}{A_1} - \frac{1}{B_1}\right)\frac{(\boldsymbol{G}_1\cdot\boldsymbol{I}_1)^2}{2}\\
&+ \left(\frac{1}{C_1} - \frac{1}{B_1}\right)\frac{(\boldsymbol{G}_1\cdot\boldsymbol{K}_1)^2}{2} + \left(\frac{1}{A_2} - \frac{1}{B_2}\right)\frac{(\boldsymbol{G}_2\cdot\boldsymbol{I}_2)^2}{2}\\
&+ \left(\frac{1}{C_2} - \frac{1}{B_2}\right)\frac{(\boldsymbol{G}_2\cdot\boldsymbol{K}_2)^2}{2}.
\end{aligned} \tag{7}$$