



# On YORP-induced spin deformations of asteroids

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## ABSTRACT

The alteration of the spin states of small Solar-System bodies by the YORP thermal effect has recently become a plausible and, for some, the favorite candidate for the formation of binary asteroids. The idea is that if an asteroid is slowly spun up to a state where some strength measure is exceeded; it can no longer remain rigid and adjusts to a new configuration. Such a process might involve global fission, global shape changes without fission, or gradual surface mass loss with subsequent mass re-accumulations forming a secondary body.

Here I analyze the changes in the shape, spin, and state during slowly increasing angular momentum of rubble-pile, self-gravitating, homogeneous ellipsoidal bodies undergoing homogeneous motions. I use, as appropriate for rubble-pile asteroids, the strength models of granular materials with zero tensile strength (cohesionless but arbitrary dilatancy); those are characterized by the “angle of friction” material constant. There are distinct limit spins depending on that angle of friction and the shape, which were previously presented [Holsapple, K.A., 2001. *Icarus* 154, 432–448; Holsapple, K.A., 2004. *Icarus* 172, 272–303]. Here the deformations and state changes when the angular momentum is slowly increased from that of a limit spin state are determined, to study the YORP processes. When a body is at its limit spin and the angular momentum increases further, the body deforms in a unique way along definite paths in the ellipsoidal shape space: it evolves as an elongating shape with an increasing rotational inertia, which in most cases produces a *decreasing* spin. I give exact analytical solutions for those shape and spin histories, as well as the histories of the mass density, angular momentum and energy. Comparison to other approaches is made.

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## 1. Introduction

The Yarkovsky–O’Keefe–Radzievskii–Paddack, “YORP” effect (Rubincam, 2000) is a process that can modify the spin of small asteroids by the absorption and re-radiation of heat from the Sun. That process has recently become a popular candidate for the formation of binary asteroids.

YORP is a very slow process, typically measured in tens of thousands, millions, or even hundreds of millions of years (Bottke et al., 2006). For example, Durech et al. (2008a) measured the rotational acceleration for the small 1.7 km Asteroid 1862 Apollo as  $5.5 \times 10^{-8} \text{ rad/d}^2$  which is about  $10^{-17} \text{ rad/s}^2$ . Durech et al. (2008b) report a probable value of  $1.15 \times 10^{-8} \text{ rad/d}^2$  for the very elongated Asteroid Geographos. The limit spin rates of asteroids are on the order of  $10^{-3} \text{ rad/s}$ , so, at that acceleration, achieving the limit spin starting from a zero spin would require a time of  $10^{14} \text{ s}$  (3 million years). Such a process is quasi-static, and characterized by a very slowly changing spin or angular momentum. That fact justifies a quasi-static analysis in which all accelerations, other than those

of a steady but slowly changing spin, are ignored. A future paper will consider more rapid spin-ups and fully dynamic processes.

There are three distinctly different approaches to this problem in the literature. The approach here is a continuum deformation analysis, which requires that any granular structure is small compared to the body size. A great advantage of this approach is that the well developed and characterized models of soil and rock mechanics used in terrestrial studies can be applied. Then the theory is governed by sets of partial differential equations: the balance of mass, momentum and energy. In principle a computer code could solve the general non-homogeneous case; that is a popular approach for bodies of any shape and for those where the self-gravity can be ignored. But no one has done that yet for a spinning, self-gravitating continuum body.

But here purely analytical solutions are presented. In order to get an analytical solution the motions are restricted to those that are uniform over the body: so-called homogeneous motions. Then the partial differential equations reduce to a set of ordinary differential equations that can be integrated. That simplification excludes processes like deformation localization, and mass shedding. But it does provide a good analytical end-case for the physics and for numerical studies. The theory here uses the same

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assumptions about the motions as in the classical literature by Chandrasekhar (1969) and others for fluid bodies, but extends those results to the soil mechanics models for solid bodies rather than just for incompressible fluids.

A very different theoretical approach is given by Scheeres (2007, 2009); he looks at the kinematics, equilibrium and stability of individual rigid bodies initially resting upon each other (as for a contact binary asteroid) but does not consider internal deformations of those pieces. Of course that assumes a binary or collection of finite bodies to start with, but his is a useful approach to determine their equilibrium, stability and subsequent evolution.

In an intermediate approach, Walsh et al. (2008) present a code numerical study using an “N-body” code of a rubble-pile asteroid being slowly spun up due to the YORP effect. The N-body approach models the asteroid as a large number ( $\sim 1000$  in that case) of vibrating spherical particles with gravitational interactions, but no inter-particle bonding.<sup>1</sup> To first order, that is an extension of rigid-body analyses to a large number of particles, but then they also model the particle interactions in a simple way using a coefficient of restitution in each contact to account for energy loss mechanisms when the spheres collide. They find that the breakup outcomes are not strongly affected by the value used for that coefficient, although its value does influence subsequent re-accumulation. This approach has the great advantage of being able to model arbitrary deformational changes including mass loss and subsequent re-accumulations, but suffers from the simplicity of the particle interaction modeling.

Walsh et al. (2008) imposed an increasing spin applied to an initially ellipsoidal body. They find that the increasing spin, of both initially spherical and initially prolate bodies,<sup>2</sup> causes the bodies to adjust their shape and become oblate, and can cause mass shedding of particles from the equator which can subsequently re-form into a bound satellite. But, as is true for any code approach, they must track each calculation as a separate problem. The analytical approach given here has the advantage of determining the results as specific algebraic functions of the problem parameters.

Holsapple (2001, 2004) previously used the continuum model for homogeneous, spinning, self-gravitating bodies and determined the quasi-static spin limits of ellipsoidal asteroids. His analyses extended to solid bodies the classical studies of the equilibrium limits for spinning fluid bodies – the well-known Maclaurin and Jacobi shapes for fluids – and those classical fluid results were obtained as a special case of his general approach. He used the well-known Mohr–Coulomb model continuum failure model of soil mechanics. Holsapple (2001, 2004) derived exact solutions for the envelopes of spin limits as a function of the ellipsoidal shape and the angle of friction of the Mohr–Coulomb failure model for the body.

Here the continuum approach results are presented for the deformation and state histories during a quasi-static, slowly increasing angular momentum of asteroids. The results found here are not the same as the Walsh et al. (2008) code results using the rubble-pile N-body approach. Here, consistent with the homogeneous motion assumption, it is found that any ellipsoidal body will adjust its shape smoothly before achieving a spin sufficient for equatorial mass loss. With imposed increasing angular momentum, initially oblate bodies (including initially spherical ones) remain oblate and become more flattened, but bodies beginning with a general ellipsoidal or prolate shape do not become oblate. And, surprisingly, it is found that the term “spin-up” is usually

not appropriate at the limit states. For spins below the limit spins, where the body remains rigid, the YORP effect can indeed increase the spin rate. But at the limit spin states where deformations are possible, the deformation elongates the body and increases the rotary inertia sufficiently that in most cases an increasing angular momentum results in a *decreasing* spin rate.

The interpretation of these results is provided in the final summary at the end of the paper.

## 2. The theory

### 2.1. Gravity, potential energy and body forces

I follow mainly the notation of Chandrasekhar (1969), used also in Holsapple (2001, 2004), together with common notation of continuum mechanics (e.g. Truesdell and Toupin, 1960). A homogeneous spinning self-gravitating ellipsoidal body has the lengths of the semi-axes labeled in order from largest to smallest as  $[a \ b \ c]$ . The aspect ratios, the ratios of the semi-axis lengths, are denoted as

$$\alpha = \frac{c}{a}, \quad \beta = \frac{b}{a} \quad (1)$$

so that always  $\alpha \leq \beta$ .

The gravitational potential is given in the  $\mathbf{x} = [x \ y \ z]^T$  coordinate system aligned with the ellipsoid's principal axes as

$$U = \pi \rho G (-A_0 + A_x x^2 + A_y y^2 + A_z z^2) \quad (2)$$

where  $G$  is the gravitational constant and  $\rho$  is the uniform mass density. The  $A_x, A_y, A_z$  terms can be given for an ellipsoidal body in terms of standard incomplete elliptic integrals (e.g. Chandrasekhar, 1969, Chapter 3, Eqs. (33)–(35)); or, in the special cases of a prolate or oblate body, in terms of elementary trigonometric forms. These gravitational terms are related by

$$A_x + A_y + A_z = 2 \quad (3)$$

$$A_x a^2 + A_y b^2 + A_z c^2 = A_0 \quad (4)$$

The three  $A_i$  (using an index notation) are non-dimensional and depend solely on the aspect ratios:  $A_i = A_i(\alpha, \beta)$ . Subsequently, using Eq. (4),  $A_0$  can be put into the dimensional form

$$A_0 = a^2 F_G(\alpha, \beta) \quad (5)$$

which defines a specific non-dimensional function  $F_G(\alpha, \beta)$  of the shape parameters. The gravity body-force components are given from the potential function in an index notation as

$$b_i = -\frac{\partial U}{\partial x_i} = -2\pi \rho G A_i x_i \quad (\text{with no sum on } i) \quad (6)$$

and integrating the  $U$  of Eq. (2) gives the potential energy of the body as

$$PE = -\frac{2}{5} \pi \rho G M A_0 \quad (7)$$

where the mass of the ellipsoidal body  $M$  is given as

$$M = \frac{4\pi}{3} \rho abc$$

During any process, balance of mass requires that

$$\frac{d}{dt}(\rho abc) = 0 \quad (8)$$

It will be useful to define a “body force” tensor  $\mathbf{B}$  by taking its matrix components in the principal axis coordinate system as

<sup>1</sup> Although there remains the interlocking of spheres providing a shear strength under increasing pressure, just as in the continuum Drucker–Prager soil model used here.

<sup>2</sup> Although purely smooth symmetrical shapes such as spheres or ellipsoids cannot have a YORP effect, slight surface roughness is sufficient to induce one (Rubincam, 2000).

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